Symbolic Cognition Symposium
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Since originally writing the questions to focus on for these one-pagers I am more convinced that the practical questions are probably of more importance to me at this stage in the development of the work of the group. Keeping a focus on the intended audience, I offer two examples from my research whilst trying to highlight how I am struggling with the pedagogical (practical) considerations of such work. As a pretext, I believe that there is a growing body of knowledge of work on students’ cognition with mathematics, particularly more “abstract” mathematics, which PME’s Advanced Mathematical Thinking working group created a great cornerstone for, but I believe the field is now encompassing new theoretical perspectives on semiotics, mathematical history, epistemology and evolution of sign systems/symbolic thinking, as well as incorporating the latest advances in representationally and dynamically rich technology environments. This surely has an impact on how “more” students “can” learn mathematics as well as how we teach it. Theory (of student thinking), history and software development are important new features but if a collective body of work on symbolic cognition is to have an impact it needs to speak to teaching in a colloquial sense to various populations. Is this possible?

Example 1. *Intersection between physical manipulatives, gesture and metaphor and the role of metacognition.*

Two undergraduates are working on a problem involving Calculus. They need to find the surface area of that part of the sphere with equation \( x^2 + y^2 + z^2 = a^2 \) that lies inside the cylinder \( x^2 + y^2 = ax = 0 \). The undergraduates eventually draw a diagram (see figure 1) but the following text offers an illustration of the types of work they were involved in to “make sense” of the problem. Note, I do not want to use the word visualize, mainly because of the language and actions they incorporate (I’ve highlighted some in red), but also the language I use (I am S) to make sense of what they are thinking:

S: And you know what a cylinder is ... I know it may seem a silly question.
J: Turn the paper around and you've got a ... that's from your ... your y-
S: Ok, now what does that...
J: axis is bang-smack in the middle of that.

[折叠纸]

And your y goes from minus a half to plus a half, and you cut your x off down there and there ...
S: So what will you be left with, with that piece of paper?
J: We'll just be left with the two arcs.
S: Has that piece of paper made it clear, that you were to chop it up.
T: Yeh, 'cos it's not solid is it?
J: No
This study was part of a larger inquiry into the students’ use of metacognition, particularly self-regulatory thinking, in advanced mathematics. Albeit simple, I think it offers an insight into the symbiotic role of perception and language to denote expression and “live” mathematical problem-solving. I see the phrases, somewhat metaphoric (?) but more so expressive, as an avenue to understand what students are thinking when they struggle with interpreting mathematical expressions and problem-statements into expressible forms or diagrams, what information they extract, what they manipulate and organize, and finally what computational expressions that they are eventually “happy” with (again see figure 1 for a schema of their work). Ok, you cannot see the whole problem-solving episode but I use it as springboard example. Also note the role of deixis (not visually demonstrable here but in part in their language use), in pointing to specific parts of the system they are trying to unpack. The focus on the imaginary rotation axis “bang-smack in the middle” {which they pointed at through the curled up paper}, the spatial relationship they have between the paper and the symbol system they are trying to work with and the pseudo-physical relationship they have with the system in terms of “cutting it up”. I do see self-regulatory thinking evident here in terms of organizing and monitoring behavior.

In practice. Well, I could claim that we need to listen more to what our students say and interpret their gestural actions and use of language. I could continue to give lots more simple examples. I could also say that we could use such examples to heighten awareness of mathematics teachers to be better prepared for mathematics instruction at the university level. But that sounds too naïve. We need to deal with an institutional belief shift first into how mathematics should be taught vs how students should learn to inquire. Even with inquiry spaces alive in university education, what do “teachers” do?

Example 2. Dynamic interactive environments and classroom connectivity
I think the use of software coupled with a hardware infrastructure that supports communication is a recent critical innovation in mathematics education. Software that not only supports dynamic, but also collaborative, interactions is a potential new system for students to not only explore and make sense of mathematical symbols but also the “invisible”, more inherited symbolic constructs that embody ideas of variation and change in algebra and geometry.

He is an example of using SimCalc MathWorlds software which deals with simulations (motions) of graphs that students can create graphically (via hot-spots), algebraically (via expression inputs) or physically (via motion imports). Multiple representations (including the motion), can be displayed,
including links between rate and accumulation graphs (see figure 2). We also exploit
graphical (more qualitative) editing by using piece-wise functions.
Here is an activity used in middle and high school as well as in upper level university
mathematics classes. I believe that our inquiry should also be concerned with pre- and in-
service education as well as traditional mathematics courses. Dealing with a re-
conceptualization of the mathematics of change and variation is sometimes more critical
to math majors (especially with a focus on school teaching) than on their own prior
cognition of the subject matter.

The activity calls the class to be split into numbered groups. You need to create a motion
(using an algebraic expression) that travels at a speed equal to your group number for 6
seconds. In a high school class, this can be in terms of Y=MX+B, and so expressions of
the form, Y=X, Y=2X, Y=3X are created. The advances of connectivity allows us to
aggregate each groups’ contributions into a common workspace. In SimCalc
MathWorlds, this is a parallel piece of software on the desktop computer that shares the
same dynamic representational features as a hand-held device version of the software.

We then ask students what they expect to see before showing them the class-aggregate.
This forces them to think about their function relative to the whole class of functions,
where the natural index for mathematical variation (i.e. slope as rate) is the index for the
natural physical set-up of the class (i.e. group number). This can force students to
generalize in ways that exploit the social set-up of the classroom. We have observed
students using gesture, deixis (studying the properties of linguistic expressions – indexes
- that cannot be interpreted without reference to a nonlinguistic context of their use, e.g.
pointing) and metaphor, in heightened ways in these “connected” environments,
propagating new forms of discourse and interaction, especially as teachers use questions
that can exploit inquiry and discussion, as well as shifting the agency from teacher-as-
answerer to teacher-as-facilitator as students argue and generalize within their own
groups and across the classroom. In answering the what do you expect to see question,
one student displayed his palm with fingers splayed out saying “it’s gonna look like a
fan” – what I call a metaphorical gesture to explain the Y=MX family of functions. A
trivial example for undergraduates but shift representations where students are just
dealing with rate graphs (e.g. velocity) and the host computer displays an aggregate of
accumulations graphs (e.g. position) then it becomes a slightly richer problem potentially
involving the Fundamental Theorem of Calculus. Consider a piecewise velocity graph (an
upside down V), and ask students what would the accumulation graph look like. Then ask
a bunch of seniors, consider you know nothing about Calculus
how would you answer
the problem? I’ve done this with some really interesting results.

Questions for practice: Can classroom connectivity, dynamic, interactive and
representationally rich software environments, change the way students perceive, become
knowledgeable and use mathematical systems for applied problem-solving, vs just solve
mathematical problems? What do we need to be aware of in using these systems? What
are best-practices in their use? Again, what forms of expressions (verbal and physical)
should we take note of/be aware of? I see these as part of Symbolic Cognition; cognition
involves our physical self in terms of verbal and gestural expression.
In summary, I made a slide for the last Symbolic Cognition meeting at PME where I diagrammatized the intersection of two spaces to understand and explain a transition to symbolic cognition. These were: i) the kinesthetic, gestural, physical-action space, and ii) social, metaphorical, discourse-action space. Intersections can be facilitated by didactics and possibly semiotic mediators (of which I use Dynamic Geometry environments as one example). This might not be useful, and too theoretical, but I would like to think more about the intersections of these two spaces, particularly from the perspective of what does this inform us about the practical implementation of such knowledge.

Truly in summary (!), do we want to know more about symbolic cognition, or do we want to know more about symbolic cognition for the benefit of others?