Semiotics, Symbols, and Mathematical Visualization

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Recently, I was called on to provide research questions that I considered to be significant and still in need of investigation, at the end of a handbook chapter recording the history of research on visualization in mathematics education over the last three decades (Presmeg, in press). Several of these questions are relevant in two of the strands that will be investigated in the Symbolic Cognition Group, namely, *Semiotics and its application to mathematics*, and *Mathematicians use of symbols and mathematical instruction*. The full list of proposed questions for research on visualization is as follows.

1. What aspects of pedagogy are significant in promoting the strengths and obviating the difficulties of use of visualization in learning mathematics?
2. What aspects of classroom cultures promote the active use of effective visual thinking in mathematics?
3. What aspects of the use of different types of imagery and visualization are effective in mathematical problem solving at various levels?
4. What are the roles of gestures in mathematical visualization?
5. What conversion processes are involved in moving flexibly amongst various mathematical registers, including those of a visual nature, thus combating the phenomenon of compartmentalization?
6. What is the role of metaphors in connecting different registers of mathematical inscriptions, including those of a visual nature?
7. How can teachers help learners to make connections between visual and symbolic inscriptions of the same mathematical notions?
8. How can teachers help learners to make connections between idiosyncratic visual imagery and inscriptions, and conventional mathematical processes and notations?
9. How may the use of imagery and visual inscriptions facilitate or hinder the reification of processes as mathematical objects?
10. How may visualization be harnessed to promote mathematical abstraction and generalization?
11. How may the affect generated by personal imagery be harnessed by teachers to increase the enjoyment of learning and doing mathematics?
12. How do visual aspects of computer technology change the dynamics of the learning of mathematics?
13. What is the structure and what are the components of an overarching theory of visualization for mathematics education? (Presmeg, in press)

All of these questions in one way or another address semiotic aspects involved in the teaching and learning of mathematics. Because Peirce in his triadic model of semiosis suggested that a sign may be of one of three forms, namely, iconic, indexical, or symbolic, the particular symbolic modes of representation that are the focus of the Symbolic Cognition enterprise may be considered to be a subset of the signs that may be used in mathematical thinking and learning (Peirce, 1998). I am particularly interested in

doing further empirical research on the fifth question in the foregoing, concerning moving amongst mathematical registers, but the first, second, seventh and eighth questions are also apposite to the work of the Symbolic Cognition group and will be addressed, thus melding the artificially separated aspects of theoretical elements and the practices of mathematics teaching and learning. I am also intending to work of the thirteenth question concerning an overarching theory of visualization for mathematics education, during sabbatical leave in spring of 2007.

In particular, the foregoing issues for research seem to relate to the following questions posed for the Symbolic Cognition group:

Strand 1

- What make mathematical signs, figures, and shapes symbolic? When something is symbolic, how is it perceived and how is it used?
- What types of mathematical ideas, concepts, or definitions do we refer to when we say something is a symbol?
- What theories from semiotics and related fields are relevant to our study, and what research methods are transferable?

Strand 2

- What are diagrams used for in mathematics?
- Are symbols cultural artifacts or are mathematical processes encapsulated in the form of symbols; if so, how?
- Are symbols idiosyncratic or can they be socially constructed; if so, how, and in what forms?

It is likely that further questions from these strands will also be implicated in my research, because I intend to conduct case studies either of students in advanced mathematics courses in high school, or of freshman or sophomore students studying mathematics in a university setting. I am particularly interested in the *problematique* of movement amongst registers in the learning of trigonometry.

References

Peirce, C. S. (1998). *The essential Peirce: Selected philosophical writings, Vol. 2 (1893-1913).* The Peirce Edition Project. Bloomington, Indiana: Indiana University Press.

Presmeg, N. C. (in press). Research on visualization in learning and teaching mathematics: Emergence from psychology. In A. Gutiérrez and P. Boero (Eds.), *Handbook of research on the psychology of mathematics education.* Dordrecht: Sense Publishers.