Symbolic Cognition in Advanced Mathematical Thinking
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The focus of this NSF project is given in the following suggested framework:

Our work (eventually a handbook on research and practice in undergraduate mathematics teaching and learning) will primarily reach teachers of u/g mathematics in 2-4 year colleges and Tier 2 universities teaching lower level courses such as algebra, geometry, Calculus, specifically instructors with no background in teaching or learning theory of symbol use and evolution of symbols systems in mathematics. Secondarily, it will serve as a major source for researchers with interests in symbol use and development in mathematics.

These two purposes inherently introduce a tension into our work, for while we may have a researcher’s eye on the direction we may take, the purpose of this project is to inform the practical needs of mathematical teachers in undergraduate mathematics.

My own long-term work focuses on how individuals develop mathematical thinking from birth to adulthood. (An outline of the theoretical development is here, with links to recent papers.) Following the trends in mathematics education through the work of Piaget, Bruner, Fischbein, van Hiele, SOLO taxonomy, and various theories of process-object construction, I have found a personal categorisation of mathematical growth which has resonances with all of these, building from perceptions, using physical interaction and thought experiments to focus on the properties of objects, actions that are performed on objects and then symbolised to give basic symbols for arithmetic computation and algebraic manipulation, through to an awareness of the properties of perceptions and actions that lead to a set-theoretic form of axiomatic definition and formal proof.

Symbolic cognition applies to the use of symbolism throughout all these areas, not just the symbolism of arithmetic and algebra (which I call proceptual symbolism because the symbols operate dually and flexibly as both process and concept) but also in the diagrams and illustrations used, the symbolism of logic, the varied use of words and so on, and the meanings and usage of these symbols for different kinds of mathematicians and students.

It illustrates that, though many mathematics teachers (including those in two year colleges) may see symbolism in a specific sense relating to, say algebra and calculus, we need a broad consensus as to the broader meaning of symbolism in our work. Highly technical formulations may (or may not) help us build our second objective as a source for researchers, but we need to use our various ways of looking at symbolic cognition to build a practical approach of help in our primary objective. The symbolism envisaged in two-year college courses is very different from that of formal axiomatic proof or of creative mathematical research. The professional mathematician needs a ready presentation of ideas in a manner that is helpful for theoretical and practical purposes.

The three strands suggested for our work each have theoretical and practical aspects: the first looking at semiotics and its applications to mathematics (which I would prefer to see widened not just to semiotic theory, but to other theories of mathematical representation
and meaning), secondly to the viewpoint of mathematicians and the practice of university teaching, and thirdly turning to the role of symbols in (dynamic) technologies. In considering these strands, we need to be constantly monitoring the creative tension between different theories and different practices in different kinds of institution to produce a useful and meaningful synthesis.