My research interest deals with the teaching and learning of mathematics through their mediation by dynamic technologies, in particular dynamic geometry (Strand 3). Dynamic geometry by introducing the dragging operation mediates the idea of variation and variable that is the essence of mathematics, as mathematical properties can be viewed as invariants amidst change. However the idea of varying is lost for most of the students in the usual algebraic symbolism or even in static representations like graphical representations of functions or table of values. Therefore we are interested in the design of teaching learning sequences for fostering learning processes starting from an embodied notion of variable and variation in a dynamic technological environment and moving to an internalization of the actions performed in the environment through the evolution of signs giving account for these notions towards socially shared signs.

The design of such teaching learning sequences is mainly based on three theoretical approaches:

- the Vygotskian approach of semiotic mediation about the mediated character of any human activity, the role of signs in the conceptualization processes as well as the contribution of interpersonal processes in the internalization processes;

- the instrumentation approach (Vérillon and Rabardel 1995) about the appropriation processes of an artifact by an individual : an artifact is not transparent and requires an appropriation by its user through a process called ‘instrumental genesis’ in which the user develops ‘schemes of utilization’ (the notion of scheme derives from the Piagetian notion as it is used in the theory of conceptual fields developed by Vergnaud 1991);

- the theory of didactic situations (Brousseau 1997) and especially the notion of “milieu”. Learning is described as emerging from exchanges between the student and a “milieu” organized with teaching (didactic) intentions:
  - Knowledge should appear as a means of producing winning strategies in problems to be solved,
  - The milieu is offering means to students of invalidating solving strategies, estimating their effectiveness and their validity.

The design of our teaching sequences is based on cycles of students’ activities in a dynamic geometry environment (Cabri-geometry) alternating with written individual reports and collective discussion in classrooms. Activities are meant for either contributing to the formation of signs or to the construction of mathematical knowledge as means of solution of a problem. Written reports are used to foster the formulation of actions (natural language or symbolic expressions) and are the basis of the collective discussion in which the teacher may introduce new words and symbols and organizes the evolution of their signified. The role of the teacher is critical for ensuring an inter-
subjective space intermediate between the individual use of signs constructed by students
and the objectified social use of signs in the mathematical culture transcending the
individuals.

Our theoretical reflection is based on two examples: the notion of function and graph of
function in high school, the notion of differential equation at university.

Teaching sequences about the notion of variable and function as co-variation were
designed, carried out with 10th grade students and observed (PhD work of Falcade,
Falcade 2003, Falcade et al. 2004). The learning aim is the construction by students of the
notions of independent and dependent variables, and the notion of function as covariation
of a dependent variable and independent variables. These notions are first experienced in
a geometric context in a dynamic geometry environment. The notion of domain, image
and covariation are constructed as schemes of utilization of the trace and drag tools
provided by the dynamic geometry environment. It is only later that the problem of
representing geometrically numerical functions is posed. The usual way of representing a
function by a graph appears thus as a solution consisting of representing a numerical
function by means of a geometric function.

The mediation role of the computer environment about the notions of first order
differential equation and of solutions has also been experimented (PhD work of Moreno).
The learning aim is the construction of a relationship between the differential equation as
providing \( y' \) as function of two independent variables \( x \) and \( y \), and the family of solutions
\( y(x) \) in which \( y \) is a variable depending of \( x \). Activities about differential equations in
Cabri-geometry were designed and observed with university students of a preservice
teacher education program (fourth university year). Although those students were
supposed to have already been taught these notions, the observation of their work
revealed how these activities destabilized them and showed the lack of relationship for
them between the algebraic symbolism and the other means of representations, as well as
the difficulty of interpreting a relationship between \( y' \), \( x \) and \( y \) valid for any point of the
Cartesian plane (viewed as a relationship in a coordinate geometry) as a differential
equation (viewed as an equation in a functional setting).

References
Cooper, R. Sutherland, & V. Warfield, Trans. and Eds.). Dordrecht: Kluwer.

Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Trans. and Eds.). Dordrecht:
Kluwer

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