Metamathematical processing (higher-order) and a touch of gesture

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higher-order thinking

- Higher-order can exist in young learners - Piaget studies of logico-mathematical reasoning; van Hiele levels (move from Level 1 - visual - to Level 3 - abstract/relational) - both following empirical experiences

- Formal deduction (Piaget) or metamathematical reasoning across systems (Level 5) - higher-order processing that can/cannot exist in AMT
Varignon’s Area ...

- a case study in multiple concept images ...
- how do you access it?
origins: declarative memory

- Declarative memory: Two types - Episodic (memory for particular times/places) and Semantic (memory for facts).
- Semantic memory is accumulated in cortical sites as a consequence of experiences.
- Episodic (or source) needs semantic and frontal lobes to store past experiences and recollect - an important feature of metacognition or simply reflection, and AMT.
- Source memory errors common in early learners and increase with age.
how do we access this?

how do we see this and make sense of student’s emerging AMT?

enter the role of gesture
development of symbolic cognition

- TRANSITIONS (along cognitive pathways) that include semiotic mediators (e.g. DGS)
- Interaction of two important spaces:
  - Kinaesthetic, gestural, physical action space
  - Social, metaphorical, discourse action space

Didactics

Transition

Symbolic Cognition
pre-history of writing in gesture

- prehistory of written language introduces a system of signs and symbols that requires an entire cultural development of the child.

- written language constitutes a system of signs that designate sounds and words of spoken language - these are signs for real entities. Gradually, the intermediate link of spoken language disappears/becomes silent - written language is converted into signs.

- What is left in reality is gestural actions to act as intermediaters between metaphor/similes and then systems of signs and formal symbol systems.
gestures ... an intro

• writing in air

• signs are fixed gestures

• Gesture as an embodiment of a situated cultural context rather than a reaction to a mental image

• On this line of thought Gesture and Indexical Retrieval and closely linked in discourse:
  Example: Towards the end of a line of discourse the “open hand” of the interlocutor is asking for cooperation

• Progression of gestural actions through the space from grounded to open to away is hierarchial – use of hands

• Body of gesture vs interactive space - what is a gesture space? Recall two action spaces previously
Issues of Representation

origin of writing signs: children’s scribbles - “pencil scratches” as gestural representation

Vygotsky’s example of children using fingers as running and leaping (deictic motion of two connected fingers) → tracking (consider ink marks) - why does a child do this? - why does an undergraduate do this? Compare with induction, Riemann integration, ODEs

Gestures as indelible MARKS on a paper

Students actions and interactions with representational system (dynamic) both physically/deictically/discursive – how much is accommodated, “borrowed” from the system and experience with the system into compact formalisms: Herbert Dreyfus - Being in the World (interpretation of Heidegger’s Work)
some mathematical examples ...
first classroom connectivity
Metaphorical Gesture (e.g. fans - Y=MX)

Linguistic Gesture (e.g. systems of equations)

Parallelism?
kinship between gestural depiction and depiction by drawing

- Find the surface area of that part of the sphere with equation \( x^2 + y^2 + z^2 = a^2 \) that lies inside the cylinder \( x^2 + y^2 - ax = 0 \).

[Answer \( 2a^2 (\pi-2) \)]
And you know what a cylinder is ... I know it may seem a silly question.

271 J: Turn the paper around and you've got a ... that's from your ... your y-

S: Ok, now what does that...

J: axis is bang-smack in the middle of that.

[Folding of paper]

And your y goes from minus a half to plus a half, and you cut your x off down there and there ...

S: So what will you be left with, with that piece of paper?

J: We'll just be left with the two arcs.

S: Has that piece of paper made it clear, that you were to chop it up.

T: Yeh, 'cos it's not solid is it?

J: No
hand-waving circulars

Walk a circle of diameter equal to three times your group number
notion of play (Vygotsky)

- problem with making the plaything unimportant
- executing representational gestures with it
- indicatory gestures (pointing, shoving) ➔ exploratory gestures (grouping, constructing) - a hierarchical physical action space
- is this a mechanism to make symbolic meaning?
- symbolic play as a complex system of speech through gesture
- play through exploration leads to discovery of an independant sign/symbol ➔ symbolic transition through met-b-for ?
- some final examples in play-space
touching Gauss

• Euclidean construction

• Dynamic geometry - not just about dragging but touching, tapping, propagating ➔ iterating

• Gesture is internal/instantiated within a technological environment. Embedded gesture

• what do we learn through construction/exploration - is it play? does it structure our proof - is it intrinsic?
regular 5-gon
regular 17-gon (heptadecagon)
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