Summary of the third meeting of the Working Group on
Symbolic Cognition in Advanced Mathematics
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Working Session 5: International Meeting of the Psychology of Mathematics

Following two meetings as a discussion group in the previous international meetings, we moved forward to work within three specifically defined sub-groups formulating existing and future lines of inquiry with related data sets. We aimed to focus on the use of data, which ranged from qualitative think-aloud accounts to pre-post tests asking what is the role of data in our study. If there is a theory of symbolic cognition then what empirical evidence do we and can we collect to support and falsify our inquiry; in particular do symbols (one might even say higher-order symbols) evolve to assist learning and reduce cognitive load or are they an active part of a functional neuropsychological model (i.e. are mathematical signs more “symbolic” or representative of how particular parts of the brain operate).

1. The use of symbols in human activity and theories of their use
We originally developed this strand of the working group to focus on more microcosmic issues of sign and symbol formation, finding parallels with theories of the evolution of symbol-systems, semiotics, semantics and semiosis, as well as how the mind understands the inter-relationship of symbols and orders relative roles of symbols in syntactic mathematical structures. We focused our attention on what is meant when something is symbolic, what makes pictures symbolic and what exactly constitutes a symbol system. Socio-cultural perspectives on the use of tools and the theory of semiotic mediation were presented and opened up the microcosmic perspective of original discussions, especially as technological environments are one kind of mediator that are beginning to transform the formation of symbolic thinking as well as what a mathematical symbol “physically” looks like in the 21st century.

The group began by looking at student work and their external use of symbol (or squiggles in an evolutionary perspective), using ideas of representation and investigating how students appear to come to know new aspects and ideas about the symbols that they had to use in their mathematical work. A more borrowed and adaptive model of cognition. Two views (or broad starting points) were highlighted ranging from the global perspective of the individual as a user/inventor of symbols vs. the socio-cultural perspective where symbols evolve as mediators of mathematical meaning and as a “shared tool” to work with scientific ideas. We returned to Peirce’s triadic relationship of signs, indices and symbols, where signs have meaning and an interpretant (in fact it is necessary for a sign to be a sign) but it was highlighted that some are presently challenging his idea of the triad being a hierarchical, referential system and that the triad can co-exist in symbol use and manipulation.
In addition, the implicit meaning signs have or have bestowed upon them needs more attention. In using signs, meaning are agreed upon, and it would be useful to study the processes involved in making such agreement from an oral tradition to an acquired, mental process. These might emerge from the situatedness of the mathematical activity.

We concluded with more questions necessary to structure the work of the group. These included the role of inner signs (mental images, internalized operations) and how does a sign make sense, or engages the brain into procedural or conceptual acts whose consequence is to retrieve an idea, process, or memory or further organize the symbolic structure at hand into something manipulable or usable. A trivial example, is the inner operation of seeing of “+” sign which leads to the recall and application of a procedure on other objects locally in a syntactic string, e.g. \(2 + 3, \mathbf{u} + \mathbf{v}, f + g\). Finally, if we talk about a symbol being part of a sign system then the first question should be what is a system, and what makes the sign system a system; an application to system theory might be useful.

2. Specific Use of Symbols by Mathematicians and Students
This group focused on how mathematicians use symbols as well as how students operate with symbols, and develop symbolic competency in advanced mathematics. More specifically how do these symbols evolve from being pedagogical artifacts, or abstract notational systems from the society of mathematicians to be part of the conscious working memory of a novice mathematician? Also, what are symbols good or bad for with reference to the work of mathematicians? Does the redundancy of the aesthetic form of symbols lead to cognitive incompetence or mathematical misconceptions, e.g. the over-use of an “x” sign in various contexts, vector calculus, group theory, etc. for different purposes.

Data of student and professional work should be collected and analyzed with respect to the effective and ineffective use of symbols in work and attempts made to track longitudinally the history of such actions and consequences. Especially when symbols have multiple meanings. We also questioned the benefits for students to have multiple meanings for single mathematical signs. We have referred to these as object-oriented interpretants that describe the many meanings that single mathematical signs carry as they move between contexts, grounded by their context and situatedness within the surrounding mathematical objects. It maybe problematic to still dichotomize the concept-symbol dualism as a mapping, or associativity between “squiggle” and mathematical process/concept vs. the physiognomy of the sign incorporating the mathematical action it is trying to embody or interpret, e.g. an integral sign.

From a higher-level cognitive perspective, the historic debate of “concepts first?” was revisited. This has a lot parallelism with the discussion of the third group and the use of dynamic, interactive environments for the purpose of developing mathematical conjectures and reasoning as a pre-cursor to the introduction and utilization of traditional symbol forms.
3. The Role of Symbol Use with Old and New Technology
This group concentrated on the development of new symbols in technological environments. The group aimed to address a variety of old and new innovations from the affordances of hand-held devices and microworlds to the computational power of Computer Algebra Systems. The group focused on the development of symbols in a technological environment and how they are generated. Are they similar? Is symbolic functioning the same? Recent research in the use of dynamic geometry systems (e.g. Cabri) have illustrated the necessity to analyze the dynamic technological environment as a semiotic mediator and the importance of classroom dialogue to construct and direct student exploration and conjecture in a collaborative classroom setting. Old and new technologies operate with different forms of symbols, which are representative of mathematical actions, or objects, but different in form. We question, what is the effect of such a rich milieu (Brousseau, 1997) of environmental actions and feedback within the technology on symbolic cognition?

We explored the nature of various technological environments that exist in mathematics education. They have variety of uses including assessment, creation/exploration of mathematical ideas as well as formal/computational processes. It is possible that these areas overlap, for example, the group presented the use of Maple for formal assessment as well as its regular computational prowess.

In each of these areas, there are symbolic/syntactical and graphical representations, and the interplay between these for various pedagogical purposes. The group questioned that if the technology is primarily syntactically driven then what is the purpose of the graphical in the realm of symbolic thinking, and similarly, what is the role of syntactical thinking/operation (and the various forms of syntax that technological environments operate in) if the graphical is the main reason or purpose of the environment.

Conclusion: The Role of Data in our Inquiry
We concluded that there are a variety of data sources for our study linked to the various theoretical perspectives we have begun to adopt from other fields as summarized above. To generalize, do we collect data for student feedback or for particular research questions that are grounded in some genre of work? Given the idea of syntax and the more grammatical aspects of symbolic thinking that has arisen in our discussion, we asked what is syntactical thinking in advanced mathematical thinking? What do we actually mean by syntactical thinking given the anomalies and inadequacies in the “mathematics as a language” metaphor that can occur in advanced mathematical symbol use/manipulation in various contexts and environments.

Our data collection is to be driven by theoretical perspectives. There are two main areas to continue to explore and begin to embed our research inquiry if we are to establish a theory of symbolic cognition in advanced mathematics. First, the semiotic perspective: When we deal with signs we should be using semiotic theories. With syntactic driven systems/language (e.g. algebra) we should use linguistic theories. Secondly, the neuro-scientific perspective: what mathematical operations are associated with how the brain
works, and how does the brain work in consolidating signs and symbols into an efficient practice that leads to competency in doing mathematics?

There are other theoretical perspectives that we have yet to commit to, for example, the socio-cultural perspective of how symbolic thinking can be an accommodation of strategies and practices from working in a society of users, with ever-more sophisticated tools being created and implemented into practice. Also, what is the role of the pedagogues in the evolution of symbolic cognition and symbolic efficacy?

Our on-going work is to finalize a working plan for a work piece that spans an international authorship on theoretical and methodological issues relating to the science of symbolic thinking. The coordinators will be proposing an overview document given our three years of work and move forward in attracting contributing authors who are committed to producing a seminal piece of work.