

## **The Role of Culture in Teaching and Learning Mathematics**

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Mathematics education there has experienced a major revolution in perceptions (cf. Kuhn, 1970) comparable to the Copernican revolution that no longer placed the earth at the center of the universe. This change has implicated beliefs about *the role of culture* in the historical development of mathematics (Eves, 1990), in the practices of mathematicians (Civil, 2002; Sfard, 1997), in its political aspects (Powell & Frankenstein, 1997), and hence necessarily in its teaching and learning (Bishop, 1988a; Bishop & Abreu, 1991; Bishop & Pompeu, 1991; Nickson, 1992). The change has also influenced methodologies that are used in mathematics education research (Pinxten, 1994). Researchers now increasingly concede that mathematics, long considered value- and culture-free, is indeed a cultural product, and hence that the role of culture-with all its complexities and contestations-is an important aspect of mathematics education.

Diverse aspects are implicated in a cultural formulation of mathematics teaching and learning. Other chapters in this volume focus more centrally on some of these aspects, for instance the following:

- the role of language and communication in mathematics education;
- equity, diversity, and learning in multicultural mathematics classrooms.

These topics will thus not be a focus in this chapter, although some issues from these fields will enter spontaneously. Topics that are central in this chapter are those arising from and extending

the notions of ethnomathematics and everyday cognition (Nunes, 1992, 1993). Various broad theoretical fields are relevant in addressing these topics. Some of the theoretical notions that are apposite are rooted in-but not confined to-situated cognition (Kirshner & Whitson, 1997; Lave & Wenger, 1991; Watson, 1998), cultural models (Holland & Quinn, 1987), notions of cultural capital (Bourdieu, 1995), didactical phenomenology (Freudenthal, 1973, 1983), and Peircean semiotics (Peirce, 1992, 1998). From this partial list, the breadth of this developing field may be recognized. This chapter does not attempt to treat the general theories in detail: The interested reader is referred to the original authors. Instead, from these fields this chapter highlights some key notions that have explanatory power or usefulness in the central focus, which is *the role of culture in learning and teaching mathematics*. The seminal work of Ascher (1991, 2002), Bishop (1988a, 1988b), D'Ambrosio (1985, 1990) and Gerdes (1986, 1988a, 1988b, 1998) on ethnomathematics is still centrally relevant and thus is treated in some detail in later sections.

In the last decade, the field of research into the role of culture in mathematics education has evolved from “ethnomathematics and everyday cognition” (Nunes, 1992), although both ethnomathematics and everyday cognition are still important topics of investigation. The developments have rather consisted in a broadening of the field, clarification and evolution of definitions, recognition of the complexity of the constructs and issues, and inclusion of social, critical, and political dimensions as well as those from cultural psychology involving valorization, identity, and agency (Abreu, Bishop, & Presmeg, 2002). This chapter in its scope cannot do full justice to political and critical views of mathematics education (see Mellin-Olsen, 1987; Skovsmose, 1994; Vithal, 2003, for a full treatment), but some of the *landscapes* from these fields-as Vithal calls them-are used in this chapter to deepen and problematize aspects of the treatment of culture in mathematics education.

This chapter has four sections. The first addresses an introduction to issues and definitions of key notions involving culture in mathematics education. The organization of the second and third sections uses the research framework of Brown and Dowling (1998). In this framework, resonating theoretical and empirical fields surround and enclose the central research topic, and the description involves layers of increasing specificity as it zooms in on details of the problematic and problems of the research issues, the empirical settings, and the results of the studies, only to zoom out again at the end in order to survey the issues in a broader field, informed by the results of the research studies examined, in order to see where further research on culture might be headed. Thus section 2 addresses theoretical fields that incorporate culture specifically in mathematics education; section 3 addresses salient empirical fields, settings, and some details of results of research on culture in mathematics education, and their implications for the teaching and learning of mathematics. Issues relating to technology and the culture of mathematics and its teaching and learning are included in this section. Using a broader perspective, the fourth (final) section collects and elaborates suggestions for possible directions for future research on culture in mathematics education.

#### Definitions and Significance of Culture in Mathematics Education

Why is it important to address definitions carefully in considering the role of culture in teaching and learning mathematics? As in other areas of reported research in mathematics education, different authors use terminology in different ways. Particularly notions such as *culture*, *ethnomathematics*, and *everyday mathematics* have been controversial; they have been contested and given varied and sometimes competing interpretations in the literature. Thus these constructs must be problematized, not only for the sake of clarification, but more importantly for the roles they have played, and for their further potential to be major focal points in mathematics

education research and practices. Further, especially in attempts to bridge the gap between the formal mathematics taught in classrooms and that used out-of-school in various cultural practices, *what counts as mathematics* assumes central importance (Civil, 1995, 2002; Presmeg, 1998a). Thus ontological aspects of the nature of mathematics itself must also be addressed.

### *Culture*

In 1988, when Bishop published his book, *Mathematical Enculturation: A Cultural Perspective on Mathematics Education* (1988a) and an article that summarized these ideas in *Educational Studies in Mathematics* (1988b, now reprinted as a classic in Carpenter, Dossey, & Koehler, 2004), the prevailing view of mathematics was that it was the one subject in the school curriculum that was value- and culture-free, notwithstanding a few research studies that suggested the contrary (e.g., Gay & Cole, 1967; Zaslavsky, 1973). Along with those of a few other authors (notably, D'Ambrosio), Bishop's ideas have been seminal in the recognition that culture plays a pivotal role in the teaching and learning of mathematics, and his insights are introduced repeatedly in this chapter.

Whole books have been written about definitions of *culture* (Kroeber & Kluckhohn, 1952). Grappling with the ubiquity yet elusiveness of culture, Lerman (1994) confronted the need for a definition but could not find one that was entirely satisfactory. Yet, as he pointed out, culture is “ordinary. It is something that we all possess and that possesses us” (p. 1). Bishop favored the following definition of culture in analyzing its role in mathematics education:

Culture consists of a complex of shared understandings which serves as a medium through which individual human minds interact in communication with one another. (1988a, p. 5, as cited by Stenhouse, 1967, p. 16)

This definition highlights the communicative function of culture that is particularly relevant in teaching and learning. However, it does not focus on the continuous renewal of culture, the

dynamic aspect that results in cultural change over time. This dynamic aspect caused Taylor (1996) to choose the “potentially transformative view” (p. 151) of cultural anthropologist Clifford Geertz (1973), for whom culture consists of “webs of significance” (p. 5) that we ourselves have spun.

This potentially transformative view assumes particular importance in the light of the necessity for negotiating social norms and sociomathematical norms in mathematics classrooms (Cobb & Yackel, 1995). The culture of the mathematics classroom, which was brought to our attention as being significant (Nickson, 1992), is not monolithic or static but continuously evolving, and different in different classrooms as these norms become negotiated. The mathematics classroom itself is one arena in which culture is contested, negotiated, and manifested (Vithal, 2003), but there are various levels of scale. Bishop’s (1988a) view of mathematics education as a social process resonates with a transformative, dynamic notion of culture. He suggested that five significant levels of scale are involved in the social aspects of mathematics education. These are the cultural, the societal, the institutional, the pedagogical, and the individual aspects (1988a, p. 14). Culture here is viewed as an all-encompassing umbrella construct that enters into all the activities of humans in their communicative and social enterprises. In addition to a view of culture in this macroscopic aspect, as in these levels of scale, culture as webs of significance may be central also in the societal, institutional, and pedagogical aspects of mathematics education considered as a social process. Thus researchers may speak of the culture of a society, of a school, or of an actual classroom. Culture in all of these levels of scale impinges on the mathematical learning of individual students.

Notwithstanding these general definitions of culture, the word with its various characterizations does not have meaning in itself. Vithal and Skovsmose (1997) illustrated this

point starkly by pointing out that interpretations of *culture* (and by implication also anthropology) were used in South Africa to justify the practices of apartheid. They extended the negative connotations still attached to this word in that context to suggest that ethnomathematics is also suspect (as suggested in the title of their article: “The End of Innocence: A Critique of ‘Ethnomathematics’”). Some aspects of their critique are taken up in later sections.

How are *culture* and *society* interrelated? The words are not interchangeable (Lerman, 1994), although connections exist between these constructs:

One would perhaps think of gender stereotypes as cultural, but of ‘gender’ as socially constructed. One would talk of the culture of the community of mathematicians, treating it as monolithic for a moment, but one would also talk, for example, of the social outcomes of being a member of that group. (p. 2)

A. J. Bishop stated succinctly, on several occasions (personal communication, e.g., July, 1985), that *society involves various groups of people, and culture is the glue that binds them together*. This informal characterization resonates with Stenhouse’s definition of *culture* as a complex of shared understandings, and also with that of Geertz, as webs of significance that we ourselves have spun. Considering the cultures of mathematics classrooms, Nickson (1994) wrote of the “invisible and apparently shared meanings that teachers and pupils bring to the mathematics classroom and that govern their interaction in it” (p. 8). Values, beliefs, and meanings are implicated in these “shared invisibles” (p. 18) in the classroom. Nickson saw socialization as a universal process, and culture as the *content* of the socialization process, which differs from one society to another, and indeed, from one classroom to another.

Part of culture as webs of significance, taken at various levels, are the prevailing notions of what counts as mathematics.

## *Mathematics*

As Nickson (1994) pointed out, “one of the major shifts in thinking in relation to the teaching and learning of mathematics in recent years has been with respect to the adoption of differing views of the nature of mathematics as a discipline” (p. 10). Nickson characterized this cultural shift as moving from a *formalist* tradition in which mathematics is absolute-consisting of “immutable truths and unquestionable certainty” (p. 11)-without a human face, to one of *growth and change*, under persuasive influences such as Lakatos’s (1976) argument that “objective knowledge” is subject to proofs and refutations and thus that mathematical knowledge has a strong social component. That this shift is complex and that both views of mathematics are held simultaneously by many mathematicians was argued by Davis and Hersh (1981). The formalist and socially mediated views of mathematics resonate with the two categories of absolutist and fallibilist conceptions discussed by Ernest (1991). Contributing the notion of mathematics as problem solving, the Platonist, the problem-solving, and the fallibilist conceptions are categories reminiscent of the teachers’ conceptions of mathematics that Alba Thompson, already in 1984, gave evidence were related to instructional practices in mathematics classrooms.

Both Civil (1990, 1995, 2002; Civil & Andrade, 2002) in her “Funds of Knowledge” project and in her later research with colleagues into ways of linking home and school mathematical practices, and Presmeg (1998a, 1998b, 2002b) in her use of ethnomathematics in teacher education and research into semiotic chaining as a means of building bridges between cultural practices and the teaching and learning of mathematics in school, described the necessity of broadening conceptions of the nature of mathematics in these endeavors. Without such broadened definitions, high-school and university students alike are naturally inclined to characterize mathematics according to what they have experienced in learning *institutional*

mathematics-more often than not as “a bunch of numbers” (Presmeg, 2002b). On the one hand, such limited views of the nature of mathematics inhibit the recognition of mathematical ideas in out-of-school practices. On the other hand, if definitions of what counts as mathematics are too broad, then the “everything is mathematics” notion may trivialize mathematics itself, rendering the definition useless. In examining the mathematical practices of a group of carpenters in Cape Town, South Africa, Millroy (1992) expressed this tension well, as follows:

[It] became clear to me that in order to proceed with the exploration of the mathematics of an unfamiliar culture, I would have to navigate a passage between two dangerous areas. The foundering point on the left represents the overwhelming notion that ‘everything is mathematics’ (like being swept away by a tidal wave!) while the foundering point on the right represents the constricting notion that ‘formal academic mathematics is the only valid representation of people’s mathematical ideas’ (like being stranded on a desert island!). Part of the way in which to ensure a safe passage seemed to be to openly acknowledge that when I examined the mathematizing engaged in by the carpenters there would be examples of mathematical ideas and practices that I would recognize and that I would be able to describe in terms of the vocabulary of conventional Western mathematics. However, it was likely that there would also be mathematics that I could not recognize and for which I would have no familiar descriptive words. (pp. 11–13)

Some definitions of mathematics that achieve a balance between these two extremes are as follows. *Mathematics* is “the language and science of patterns” (Steen, 1990, p. iii). Steen’s definition has been taken up widely in reform literature in the USA (National Council of Teachers of Mathematics [NCTM], 1989, 2000). Opening the gate to recognizing a human origin of mathematics, Saunders MacLane called mathematics “the study of formal abstract structures

arising from human experience” (as cited in Lakoff, 1987, p. 361). According to Ada Lovelace, mathematics is the systematization of relationships (as described by Noss, 1997). All of these definitions strike some sort of balance between the human face of mathematics and its formal aspects. Going beyond Steen’s well-known pattern definition in the direction of stressing abstraction, in a critique of ethnomathematics, Thomas (1996) defined mathematics as “the science of detachable relational insights” (p. 17). He suggested a useful distinction between *real mathematics* (as characterized in his definition), and *proto-mathematics* (the category in which he placed ethnomathematics). In the next section, Barton’s (1996) characterization of ethnomathematics, which resolves many of these issues and clarifies this dualism, is presented along with some evolving definitions of ethnomathematics. (For details, the reader should consult Barton’s original article.)

### *Ethnomathematics*

What is *ethnomathematics*? In his illuminating article, Barton (1996) wrote as follows:

In the last decade, there has been a growing literature dealing with the relationship between culture and mathematics, and describing examples of mathematics in cultural contexts. What is not so well-recognised is the level at which contradictions exist within this literature: contradictions about the meaning of the term ‘ethnomathematics’ in particular, and also about its relationship to mathematics as an international discipline. (p. 201)

Barton pointed out that difficulties in defining ethnomathematics relate to three categories: epistemological confusion, “problems with the meanings of words used to explain ideas about culture and mathematics” (p. 201); philosophical confusion, the extent to which mathematics is regarded as universal; and confusion about the nature of mathematics. The nature of mathematics is part of its ontology, and because both ontology and epistemology are branches of philosophy,

all of these categories may be regarded as philosophical difficulties. The strength of Barton's resolution of the difficulties lies in his creation of a preliminary framework (he admitted that it might need revision) whereby the differing views can be seen in relation to each other. His triadic framework is an "Intentional Map" (p. 204) with the three broad headings of *mathematics*, *mathematics education*, and *society* (cf. the whole day of sessions dedicated to these broad areas at the 6th International Congress on Mathematical Education held in Budapest, Hungary, in 1988). The seminal writers whose definitions of ethnomathematics he considered in detail and placed in relation to this framework were Ubiratan D'Ambrosio in Brazil, Paulus Gerdes in Mozambique, and Marcia Ascher in the USA.

As Barton (1996) pointed out, D'Ambrosio's prolific writings on the subject of ethnomathematics have influenced the majority of writers in this area. Thus on the Intentional Map, although D'Ambrosio's work (starting with his 1984 publication) falls predominantly in the socio-anthropological dimension between *society* and *mathematics*, some aspects of his concerns can be found in all of the dimensions. In his later work, he increasingly used his model to analyze "the way in which mathematical knowledge is colonized and how it rationalizes social divisions within societies and between societies" (Barton, 1996, p. 205). In his early writing, D'Ambrosio (1984) defined ethnomathematics as the way different cultural groups mathematize-count, measure, relate, classify, and infer. His definition evolved over the years, to include a changing form of knowledge manifest in practices that change over time. In 1985, he defined ethnomathematics as "the mathematics which is practiced among identifiable cultural groups" (p. 18). Later, in 1987, his definition of ethnomathematics was "the codification which allows a cultural group to describe, manage, and understand reality" (Barton, 1996, p. 207).

D'Ambrosio's (1991) well-known etymological definition of ethnomathematics is given in full in the following passage.

The main ideas focus on the concept of *ethnomathematics* in the sense that follows. Let me clarify at the beginning that this term comes from an etymological abuse. I use *mathema(ta)* as the action of explaining and understanding in order to transcend and of managing and coping with reality in order to survive. Man has developed throughout each one's own life history and throughout the history of mankind *techné's* (or *tics*) of *mathema* in very different and diversified cultural environments, i.e., in the diverse *ethno's*. So, in order to satisfy the drive towards survival and transcendence in diverse cultural environments, man has developed and continuously develops, in every new experience, **ethno-mathema-tics**. These are *communicated* vertically and horizontally in time, respectively throughout history and through conviviality and education, relying on memory and on sharing experiences and knowledges. For the reasons of being more or less effective, more or less powerful and sometimes even for political reasons, some of these different *tics* have lasted and spread (ex.: counting, measuring) while others have disappeared or been confined to restricted groups.

This synthesizes my approach to the history of ideas. (p. 3)

As in some of his other writings (1985, 1987, 1990), D'Ambrosio is in this definition characterizing ethnomathematics as a dynamic, evolving system of knowledge-the "process of knowledge-making" (Barton, 1996, p. 208), as well as a research program that encompasses the history of mathematics.

Returning to Barton's Intentional Map, the work of Paulus Gerdes is "practical, and politically explicit," concentrated in the *mathematics education* area of the Map (Barton, 1996, p. 205). Gerdes's definition of ethnomathematics evolved from the mathematics implicit or

“frozen” in the cultural practices of Southern Africa (1986), to that of a mathematical movement that involves research and anthropological reconstruction (1994). The work of mathematician Marcia Ascher (1991, 1995, 2002), while overlapping with that of Gerdes to some extent, falls closer to the *mathematics* area on the Map, concerned as it is with cultural mathematics. Her definition is that ethnomathematics is “the study and presentation of the mathematical ideas of traditional peoples” (1991, p. 188). When Ascher (1991) worked out the kinship relations of the Warlpiri, say, in mathematical terms, she acknowledged that she was using her familiar “Western” mathematics. In that sense her ethnomathematics is subjective: The Warlpiri would be unlikely to view their kinship system through her lenses. Referring to mathematics and ethnomathematics, she stated, “They are both important, but they are different. And they are linked” (Ascher & D’Ambrosio, 1994, p. 38). In this view, there is no need to view ethnomathematics as “proto-mathematics” (Thomas, 1996), because it exists in its own right.

Finally, Barton (1996) found a useful metaphor to sum up the similarities and differences between the views of ethnomathematics held by these three proponents: “For D’Ambrosio it is a window on knowledge itself; for Gerdes it is a cultural window on mathematics; and for Ascher it is the mathematical window on other cultures” (p. 213). These three windows are distinguished by the standpoint of the viewer, and by what is being viewed, in each case. Although not eliminating the duality of ethnomathematics as opposed to mathematics (of mathematicians), these three distinct windows represent approaches each of which has something to offer. Taken together, they contribute a broadened lens on the role of culture in teaching and learning mathematics.

Several other writers in the field of ethnomathematics have acknowledged the need and attempted to define ethnomathematics. Scott (1985) regarded ethnomathematics as lying at the

confluence of mathematics and cultural anthropology, “mathematics in the environment or community,” or “the way that specific cultural groups go about the tasks of classifying, ordering, counting, and measuring” (p. 2). Several definitions of ethnomathematics highlight some of the “environmental activities” that Bishop (1988a) viewed as universal, and also “necessary and sufficient for the development of mathematical knowledge” (p. 182), namely counting, locating, measuring, designing, playing, and explaining. One further definition brings back the problem, hinted at in the foregoing account, of *ownership* of ethnomathematics. Whose mathematics is it?

Ethnomathematics refers to any form of cultural knowledge or social activity characteristic of a social and/or cultural group, that can be recognized by other groups such as ‘Western’ anthropologists, but not necessarily by the group of origin, as mathematical knowledge or mathematical activity. (Pompeu, 1994, p. 3)

This definition resonates with Ascher’s, without fully solving the problem of ownership. The same problem appears in definitions of everyday mathematics, considered next.

### *Everyday Mathematics*

Following on from the description of *everyday cognition* (Nunes, 1992, 1993) and important early studies that examined the use of mathematics in various practices, such as mathematical cognition of candy sellers in Brazil (Carraher, Carraher, & Schliemann, 1985; Saxe, 1991), constructs and issues are still being questioned. In this area, too, clarification of definitions is being sought, along with deeper consideration of the scope of the issues and their potential and significance for the classroom learning of mathematics.

Brenner and Moschkovich (2002) raised the following questions.

What do we mean by *everyday mathematics*? How is everyday mathematics related to *academic mathematics*? What particular everyday practices are being brought into

mathematics classrooms? What impact do different everyday practices actually have in classroom practices?” (p. v)

In a similar vein, and with the benefit of 2 decades of research experience in this area, Carraher and Schliemann (2002) examined how their perceptions had evolved, as they explored the topic of their chapter, “Is Everyday Mathematics Truly Relevant to Mathematics Education?”

All the authors of chapters in the monograph edited by Brenner and Moschkovich (2002) in one way or another set out to explore these and related questions. Several of these authors pointed out that it is problematic to oppose everyday and academic mathematics, for several reasons. For one thing, for mathematicians academic mathematics *is* an everyday practice (Civil, 2002; Moschkovich, 2002a). For another, studies of everyday mathematical practices in workplaces reveal a complex interplay with sociocultural and technological issues (FitzSimons, 2002). In the automobile production industry, variations in the mathematical cognition required of workers have less to do with the job itself than with the decisions of management concerning production procedures and organization of the workplace. Highly skilled machinists display spatial and geometric knowledge that goes beyond what is commonly taught in school: In contrast, assembly-line workers and some machine operators find few if any mathematical demands in their work, which is deliberately stripped of the need for decisions involving knowledge of mathematics beyond elementary counting (Smith, 2002). The complex relationship between use of technology and the demand for mathematical thinking in the workplace is a theme that is explored in a later section of this chapter.

Another aspect that is again apparent in all the chapters of Brenner and Moschkovich’s monograph is the importance of perceptions and beliefs about the nature of mathematics, both in the microculture of classroom practices (Brenner, 2002; Masingila, 2002) and in the broader

endeavor to bridge the gap between mathematical thinking in and out of school (Arcavi, 2002; Civil, 2002; Moschkovich, 2002a). An essential element in all of these studies is the concern to connect knowledge of mathematics in and out of school. (This issue is revisited later in this chapter.) Because of the difficulties surrounding the construct *everyday mathematics* the terminology that will be adopted in this chapter follows Masingila (2002), who referred to in-school and out-of-school mathematics practices (p. 38).

The developments described in this section parallel the genesis of the movement away from purely psychological cognitive and behavioral frameworks for research in mathematics education, towards cultural frameworks that embrace sociology, anthropology, and related fields, including political and critical perspectives. The following section introduces some relevant theoretical issues and lenses that have been used to examine some of these developments.

#### Theoretical Fields That Incorporate Culture in Mathematics Education

The notion of theoretical and empirical fields is drawn from Brown and Dowling (1998) and provides a useful framework for characterizing components of research. This section addresses some theoretical fields pertinent to culture in the teaching and learning of mathematics. Their instantiation in empirical studies is described in the next section. The reader is reminded again to consult the original authors for a full treatment of theoretical fields that are introduced in this section, which has as its purpose a wide but by no means exhaustive view of the scope of theories that are available for work in this area.

As suggested in Barton's (1996) sense-making article introduced in the previous section, in the last 2 decades there has already been considerable movement in theoretical fields regarding the interplay of culture and mathematics. One such movement is discernible in the definitions of ethnomathematics given by D'Ambrosio, Gerdes, and Ascher, as their theoretical

formulations moved from more static definitions of ethnomathematics as the mathematics of different cultural groups, to characterizations of this field as an anthropological research program that embraces not only the history of mathematical ideas of marginalized populations, but the history of mathematical knowledge itself (see previous section). D'Ambrosio (2000, p. 83) called this enterprise *historiography*.

### *Historiography*

Moving beyond earlier theoretical formulations of ethnomathematics, its importance as a catalyst for further theoretical developments has been noted (Barton, 1996). D'Ambrosio played a large role and served as advisory editor in the enterprise that resulted in Helaine Selin's (2000) edited book, *Mathematics Across Cultures: The History of Non-Western Mathematics*. The chapters in this book are global in scope and record the mathematical thinking of cultures ranging from those of Iraq, Egypt, and other predominantly Islamic countries; through the Hebrew mathematical tradition; to that of the Incas, the Sioux of North America, Pacific cultures, Australian Aborigines, mathematical traditions of Central and Southern Africa; and those of Asia as represented by India, China, Japan, and Korea. As can be gleaned from the scope of this work, D'Ambrosio's original concern to valorize the mathematics of colonized and marginalized people (cf. Paulo Freire's *Pedagogy of the Oppressed* in 1970/1997) has broadened to encompass a movement that is both archeological and historical in nature, based on the theoretical field of "historiography" and visions of world knowledge through the "sociology of mathematics" (D'Ambrosio, 2000, pp. 85–87).

Although these antedated D'Ambrosio's program, earlier studies such as Claudia Zaslavsky's (1973) report on the counting systems of Africa and Glendon Lean's (1986) categorization of those of Papua New Guinea (see also Lancy, 1983), could also be thought of as

historiography, as could anthropological research such as that of Pinxten, van Dooren, and Harvey (1983) who documented Navajo conceptions of space. Also in the cultural anthropology tradition, Crump's (1994) research on the anthropology of numbers is another fascinating example of historiography. More recent studies such as some of those collected as *Ethnomathematics* in the book by Powell and Frankenstein (1997) and the work of Marcia Ascher (1991, 1995, 2002) also fall into this category. Many of the cultural anthropological studies of various mathematical aspects of African practices, such as work on African fractals (Eglash, 1999); *Iusona* of Africa (Gerdes, 1997); and women, art, and geometry of Africa (Gerdes, 1998), may also be regarded as historiography. As part of ethnomathematics conceived as a research program, this ambitious undertaking of historiography is designed to address lacunae in the literature on the history of mathematical thought through the ages. Much of the work of members of the International Study Group on Ethnomathematics (founded in 1985), and of the North American Study Group on Ethnomathematics (founded in 2003)-including research by Lawrence Shirley, Daniel Orey, and many others-could be placed in the category of historiography (see the list of some of the available web sites following the references).

Because historiography addresses some of the world's mathematical systems that have been ignored or undervalued in mathematics classrooms, it reminds us that there is *cultural capital* involved in the power relations associated with access in school mathematics. Some relevant theories are outlined in the following paragraphs.

### *Cultural Capital and Habitus*

The usefulness of the theoretical field outlined by Bourdieu (1995) for issues of culture in the learning of mathematics has been indicated in research on learner's transitions between mathematical contexts (Presmeg, 2002a). Resonating with D'Ambrosio's original issues of

concern (although D'Ambrosio did not use this framework), Bourdieu's work belongs in the field of sociology. The relevance of this work consists in "the innumerable and subtle strategies by which words can be used as instruments of coercion and constraint, as tools of intimidation and abuse, as signs of politeness, condescension, and contempt" (Bourdieu, 1995, p. 1, editor's introduction). This theoretical field serves as a useful lens in examining empirical issues related to the social inequalities and dilemmas faced by mathematics learners as they move between different cultural contexts, for example in the transitions experienced by immigrant children learning mathematics in new cultural settings (Presmeg, 2002a). This field embraces Bourdieu's notions of *cultural capital*, *linguistic capital*, and *habitus*. Bourdieu (1995) used the ancient Aristotelian term *habitus* to refer to "a set of dispositions which incline agents to act and react in certain ways" (p.12). Such dispositions are part of culture viewed as a set of shared understandings. Various forms of capital are *economic capital* (material wealth), *cultural capital* (knowledge, skills, and other cultural acquisitions, as exemplified by educational or technical qualifications), and *symbolic capital* (accumulated prestige or honor) (p. 14). *Linguistic capital* is not only the capacity to produce expressions that are appropriate in a certain social context, but it is also the expression of the "correct" accent, grammar, and vocabulary. The *symbolic power* associated with possession of cultural, symbolic, and linguistic capital has a counterpart in the *symbolic violence* experienced by individuals whose cultural capital is devalued. Symbolic violence is a sociological construct. In that capacity it is a powerful lens with which to examine actions of a group and ways in which certain types of knowledge are included or excluded in what the group counts as knowledge (for examples embracing the learning of mathematics, see the chapters in Abreu, Bishop, & Presmeg, 2002).

### *Borderland Discourses*

The notion of symbolic violence leaves a possible theoretical gap relating to the ways in which individuals choose to construct, or choose *not* to construct, particular knowledge-mathematical or otherwise. Bishop (2002a) gave examples both of immigrant learners of mathematics in Australia who chose not to accept the view of themselves as constructed by their peers or their teacher-and of others who chose to accept these constructions. One student “shouted back” when peers in the mathematics class shouted derogatory names.

*Discourse* is a construct that is wider than mere use of language in conversation (Philips, 1993; Wood, 1998): It embraces all the aspects of social interaction that come into play when human beings interact with one another (Dörfler, 2000; Sfard, 2000). The notion of *Discourses* formulated by Gee (1992, his capitalization), and in particular his extension of the construct to *borderland Discourses*, those “community-based secondary Discourses” situated in the “borderland” between home and school knowledge (p. 146), are in line with Bourdieu’s ideas of habitus and symbolic violence. Borderland Discourses take place in the borderlands between primary (e.g., home) and secondary (e.g., school) cultures of the diverse participants in social interactions. In situations where the secondary culture (e.g., that of the school) is conceived as threatening because of the possibility of symbolic violence there, the borderland may be a place of solidarity with others who may share a certain habitus. These ideas go some way towards closing the theoretical gap in Bourdieu’s characterization of symbolic violence by raising some issues of individuals’ choices, because individuals choose the extent to which they will participate in various forms of these Discourses (see also Bishop’s, 2002b, use of Gee’s constructs).

Gee's work was in the context of second language learning but is also useful in the analysis of meanings given to various experiences by mathematics learners in cultural transition situations. Bishop (2002a) used this theoretical field in moving from the notion of cultural conflict to that of cultural mediation, in analyzing these experiences of learners of mathematics. If one considers the primary Discourse of school mathematics learners to be the home-based practices and conversations that contributed to their socialization and enculturation (forming their habitus) in their early years, and continuing to a greater or lesser extent in their present home experiences, then the secondary Discourse, for the purpose of learning mathematics, could be designated as the formal mathematical Discourse of the established discipline of mathematics. The teacher is more familiar with this Discourse than are the students and thus has the responsibility of introducing students to this secondary Discourse. As students become familiar with this field, their language and practices may approximate more closely those of the teacher. But in this transition the borderland Discourse of interactive classroom practices provides an important mediating space.

The enculturating role of the mathematics teacher was suggested in the foregoing account. However, as Bishop (2002b) pointed out, the learning of school mathematics is frequently more of an acculturation experience than an enculturation. The difference between these anthropological terms is as follows. *Enculturation* is the induction, by the cultural group, of young people into their own culture. In contrast, *acculturation* is "the modification of one's culture through continuous contact with another" (Wolcott, 1974, p. 136, as quoted in Bishop, 2002b, p. 193). The degree to which the culture of mathematics, as portrayed in the mathematics classroom, is viewed as their own or as a foreign culture by learners would determine whether their experiences there would be of enculturation or acculturation.

## *Cultural Models*

Allied with Gee's (1992) notion of different Discourses is his construct of *cultural models*. This construct, defined as "'first thoughts' or taken for granted assumptions about what is 'typical' or 'normal'" (Gee, 1999, p. 60, quoted in Setati, 2003a, p. 153), was used by Setati (2003a) as an illuminating theoretical lens in her research on language use in South African multilingual mathematics classrooms. Already in 1987, D'Andrade defined a cultural model-which he also called a *folk model*-as "a cognitive schema that is intersubjectively shared by a social group," and he elaborated, "One result of intersubjective sharing is that interpretations made about the world on the basis of the folk model are treated as if they were obvious facts about the world" (p. 112). The transparency of cultural models may help to explain why mathematics was for so long considered to be value- and culture-free. The well-known creativity principle of making the familiar strange and the strange familiar (e.g., De Bono, 1975) is necessary for participants to become aware of their implicit cultural beliefs and values, which is why the anthropologist is in a position to identify the beliefs that are invisible to many who are within the culture.

In the context of mathematics learning in multilingual classrooms, Adler (1998, 2001) pointed out aspects of the use of language as a cultural resource that relate to the transparency of cultural models. Particularly in classrooms where the language of instruction is an *additional language*-not their first language-for many of the learners (Adler, 2001), teachers must at times focus on the language itself, in which case the artifact of language no longer serves as a "window" of transparent glass through which to view the mathematical ideas (Lave & Wenger, 1991). In this case the language used is no longer *invisible*, and the focus on the language itself may detract from the conceptual learning of the mathematical content (Adler, 2001).

### *Valorization in Mathematical Practices*

If transparency of culture necessitates making the familiar strange before those sharing that culture become aware of the lenses through which they are viewing their world, then this principle points to a reason for the neglect of issues of valorization in the mathematics education research community until Abreu's (1993, 1995) research brought the topic to the fore. The *value* of formal mathematics as an academic subject was for so long taken for granted that it became a *given* notion that was not culturally questioned. Especially in its role as a gatekeeper to higher education, this status in education is likely to continue. But if ethnomathematics as a research program is to have a legitimate place in broadening notions both of what counts as mathematics and of which people have originated these forms of knowledge, then issues of valorization assume paramount importance.

Working from the theoretical fields of cultural psychology and sociocultural theory, Abreu and colleagues investigated the effects of valorization of various mathematical practices on Portuguese children (Abreu, Bishop, & Pompeu, 1997), Brazilian children (Abreu, 1993, 1995), and British children from Anglo and Asian backgrounds (Abreu, Cline, & Shamsi, 2002). As confirmed also in the research of Gorgorio, Planas, and Vilella (2002), many of these children denied the existence of, or devalued, mathematics as used in practices that they associated with their home- or out-of-school settings.

*Valorization*, the social process of assigning more value to certain practices than to others, is closely allied to Wertsch's (1998) notion of *privileging*, defined as "the fact that one mediational means, such as social language, is viewed as being more appropriate or efficacious than another in a particular social setting" (Wertsch, 1998, p. 64, in Abreu, 2002, p. 183). Abreu (2002) elaborated as follows.

From this perspective, cultural practices become associated with particular social groups, which occupy certain positions in the structure of society. Groups can be seen as mainstream or as marginalised. In a similar vein individuals who participate in the practices will be given, or come to construct, identities associated with certain positions in these groups. The social representation enables the individual and social group to have access to a 'social code' that establishes relations between practices and social identities. (p. 184)

Thus Abreu argued strongly that in the learning of mathematics, valorization operates not only on the societal plane but also on the personal plane, because it impacts the construction of social identities. At this psychological level, the construction of mathematical knowledge may be subordinated to the construction of social identity by the individual learner in cases of cultural conflict, as suggested by Presmeg (2002a).

Abreu's ideas are embedded in the field of cultural psychology. Also taking the individual and society into account, another field that has grown in influence in mathematics education research in recent decades is that of situated cognition.

### *Situated Cognition*

The theoretical field of situated cognition explores related aspects of the interplay of knowledge on the societal and psychological planes. Hence it has been a useful lens in research that takes culture into account in mathematical thinking and learning (Watson, 1998). As Ubiratan D'Ambrosio by his writings founded and influenced the field of ethnomathematics, so Jean Lave in analyzing and reporting her anthropological research has influenced the theoretical field of situated cognition. From her early theorizing following ethnography with tailor's apprentices in Liberia (Lave, 1988) to her more recent writings, following research with grocery shoppers and weight-watchers (Lave, 1997), the notion of transfer of knowledge through

abstraction in one context, and subsequent use in a new context, was questioned and problematized. In collaboration with Etienne Wenger, her theorizing led to the notion of cognitive apprenticeship and *legitimate peripheral participation* (Lave & Wenger, 1991; Wenger, 1998). In this view, learning consists in a centripetal movement of the apprentice from the periphery to the center of a practice, under the guidance of those who are already masters of the practice. This theory was not originally developed in the context of or for the purpose of informing mathematics education. However, in its challenge to the cognitive position that abstract learning of mathematics facilitates transfer and that this knowledge may be readily applied in other situations than the one in which it was learned (not born out by empirical research), the theory has been powerful and influential. The research studies inspired by Lave and reported by Watson (1998) bear witness to this strong influence.

In her later writings, Lave attempted to bring the theory of socially situated knowledge to bear on the classroom teaching and learning of mathematics. But issues of intentionality and recontextualization separate apprenticeship and classroom situations, although enough commonality exists in the two situations for both legitimately to be called *practices* (Lerman, 1998). Mathematics learners in school are not necessarily aiming to become either mathematicians or mathematics teachers. As Lerman pointed out in connection with the issue of voluntary and nonvoluntary participation, students' presence in the classroom may be nonvoluntary, creating a very different situation from that of apprenticeship learning, and calling into question the assumption of a goal of movement from the periphery to the center. At the same time, the teacher has the intention to teach her students mathematics, notwithstanding Lave's claim that teaching is not a precondition of learning. The learning of mathematics is the goal of the enterprise, and teaching is the teacher's job. In contrast, in the apprenticeship situation the

learning is not a goal in itself, for example, in the case of the tailors the goal is to make garments efficiently. Thus, as Adler (1998) suggests, “It is in the understanding of the aims of school education that Lave and Wenger’s seamless web of practices entailed in moving from peripheral to full participation in a community of practice is problematic” (p. 174).

Although the theory of legitimate peripheral participation may not translate easily into the classroom teaching and learning of mathematics, the view of learning as a social practice has powerful implications for this learning and has been an influence in changes that have taken place in the practice of teaching mathematics, such as an increased emphasis on communication (NCTM, 1989, 2000) in the mathematics classroom. Even more, situated cognition as a theory has given a warrant to attempts to bridge the gap between in- and out-of-school mathematical practices.

#### *Use of Semiotics in Linking Out-Of-School and In-School Mathematics*

In the USA, the *Principles and Standards for School Mathematics* (NCTM, 2000) continued the earlier call (NCTM, 1989) for teachers to make connections, in particular between the everyday practices of their students and the mathematical concepts that are taught in the classroom. But various theoretical lenses of situated cognition (Lave & Wenger, 1991; Kirshner & Whitson, 1997) remind us that these connections can be problematic. There are at least three ways in which the activities of out-of-school practices differ from the mathematical activities of school classrooms (Walkerdine, 1988), as follows.

- The goals of activities in the two settings differ radically.
- Discourse patterns of the classroom do not mirror those of everyday practices.

- Mathematical terminology and symbolism have a specificity that differs markedly from the useful qualities of ambiguity and indexicality (interpretation according to context) of terms in everyday conversation.

A semiotic framework that uses chains of signification (Kirshner & Whitson, 1997) has the potential to bridge this apparent gap through a process of chaining of signifiers in which each sign “slides under” the subsequent signifier. In this process, goals, discourse patterns, and use of terms and symbols all move towards that of classroom mathematical practices in a way that has the potential to preserve essential structure and some of the meanings of the original activity.

This theoretical framework resonates with that of *Realistic Mathematics Education* (RME) developed by Freudenthal, Streefland, and colleagues at the Freudenthal Institute (Treffers, 1993). *Realistic* in this sense does not necessarily mean out-of-school in the real world: The term refers to problem situations that learners can imagine (van den Heuvel-Panhuizen, 2003). A theory of semiotic chaining in mathematics education resonates with RME in that the starting points for the learning are realistic in this sense. But more specifically the chaining model is a useful tool for linking out-of-school mathematical practices with the formal mathematics of school classrooms (an example of such use is presented in the next paragraph).

In brief, the theory of semiotic chaining used in mathematics education research, as it was initially presented by Whitson (1994, 1997), Cobb, Gravemeijer, Yackel, McClain, & Whitenack (1997), and Presmeg (1997, 1998b), followed the usage of Walkerdine (1988). Although she was working in a poststructural paradigm, Walkerdine found the Swiss structural approach of Saussure, as modified by Lacan, useful in building chains of signifiers to link a home practice, such as that of a daughter and her mother pouring drinks for guests, with more formal learning of

mathematics, in this case the system of whole numbers. Walkerdine's chain had the following structure.

Signified 1: the actual people coming to visit;

Signifier 1 (*standing for* signified 1): the names of the people.

Signifier 1 and signified 1 form **sign 1**.

The daughter is asked to raise a finger for each name.

Signified 2: the names of the guests;

Signifier 2: the raised fingers.

Signifier 2 and signified 2 form **sign 2**, the second link.

The daughter is asked to count her raised fingers.

Signified 3: the raised fingers;

Signifier 3: the numerals, one two, three, four, five.

Signifier 3 and signified 3 form **sign 3**, the third link.

In this process, signifier 3 actually comes to stand for all of the preceding links in the chain: Five guests are coming to tea. Note that each signifier in turn becomes the next signified, and that at any time any of the links in the chain may be revisited conceptually. As mother and daughter move through the links of the chain, the discourse shifts successively from actual people to their names, to the fingers of one hand, and finally to the more abstract discourse of the numerals of the whole number counting system.

In her distinctive style, Adler (1998) characterized the associated recontextualizing process as that of crossing a discursive bridge:

That there is a bridge to cross between everyday and educated discourses is at the heart of Walkerdine's (1988) argument for 'good teaching' entailing chains of signification in the classroom where everyday notions have to be prised out of their discursive practice and situated in a new and different discursive practice. (p. 174)

The theory may be summarized as follows.

In his *semiology* Saussure defined the sign as a combination of a "signified" together with its "signifier" (Saussure, 1959; Whitson, 1994, 1997). Lacan inverted Saussure's model, which gave priority to the *signified* over the signifier, to stress the *signifier* over the signified, and thus to recognize "far ranging autonomy for a dynamic and continuously productive play of signifiers that was not so easily recognized when it was assumed tacitly that a signifier was somehow constrained under domination by the signified" (Whitson, 1994, p. 40). This formulation allows for the chaining process in which a signifier in a previous sign-combination becomes the signified in a new sign-combination, and so on. The chain has at its final link some mathematical concept that may be taught in school.

Using this process, a teacher can use the chain as an instructional model that develops a mathematical concept starting with an out-of-school situation and linking it in a number of steps with formal school mathematics (Hall, 2000; Hall & Presmeg, 2000). Building on the work of Presmeg (1997, 1998b) in his dissertation research, Hall taught three 4th-grade teachers to build semiotic chains appropriate to the practices of their students and the instructional needs of their individual classrooms. Using a similar semiotic chaining model, Cobb et al. (1997) reported on the emergence (using their *emergent* theoretical perspective) of chains of signification in one 1st-grade classroom. In addition to these sources, examples of mathematical chaining at elementary school, high school, and college levels may be found in Presmeg's (1997, 1998b, in press)

writings. In later research on building bridges between out-of-school and in-school mathematics, Presmeg (in press) preferred to use a nested triadic model based on the writings of Charles Sanders Peirce (1992, 1998). In addition to an *object* (which could be an abstract concept) and a *representamen* that stands for the object in some way, each nested sign has a third component that Peirce called the *interpretant*. This triadic model explicitly allows for learners' individual construction of meaning (through the interpretant) in a way that linear chains of signification can do only implicitly.

In closing this section, three more of Peirce's constructs that are relevant to the role of culture and historiography in mathematics education are introduced, for the purpose of showing the potential of these theoretical notions to provide a foundation for ethnomathematics characterized as a research program.

*Peirce's Notions of Synechism, Community, and Commens in Recognizing the Role of Culture in Mathematics Education*

In this final part of the section, I want to return to ideas in the beginning of this section by introducing some Peircean constructs that have relevance in the *historiography* of D'Ambrosio (2000) with which this section started. Peirce's (1992, 1998) philosophical writings are useful in attending to the historical development of mathematical thought and thus are relevant to the role of culture in mathematics education (Presmeg, 2003). In particular, the construct of *community*-which he used without definition-and his definitions of *commens* and *synechism*, including his "law of mind," are apposite to explorations into the place of culture in mathematical historiography. In the light of these constructs (discussed in the following paragraphs), the role of community in the public institution of mathematics education is an issue of fundamental practical importance. The significance of the community of thinkers in the evolution of

mathematical knowledge is indicated in Peirce's (1992) somewhat negative designation of the individual-uninformed by the sociocultural milieu-as ignorant and in error: "The individual man, since his separate existence is manifested only by ignorance and error, so far as he is anything apart from his fellows, and from what he and they are to be, is only a negation" (p. 55).

With regard to the genesis and evolution of mathematics, a point that has relevance in Peirce's epistemology is the continuity of past, present, and future. Continuity is central in Peirce's definition of *synechism* as "the tendency to regard continuity ... as an idea of prime importance in philosophy" (Peirce, 1992, p. 313). Synechism involves the startling notion that knowledge in its real essence depends on *future* thought and how it will evolve in the community of thinkers:

Finally, as what anything really is, is what it may finally be come to be known to be in the ideal state of complete information, so that reality depends on the ultimate decision of the community; so thought is what it is, only by virtue of its addressing a future thought which is in its value as thought identical with it, though more developed. In this way, the existence of thought now, depends on what is to be hereafter; so that it has only a potential existence, dependent on the future thought of the community. (Peirce, 1992, pp. 54–55)

Whether "the ideal state of complete information" is ever an attainable goal is a matter of doubt, but the relevance of synechism for the history of mathematics lies in the role attributed to future generations of thinkers in assessing the achievements of the past and present. The notion of synechism is further explicated in connecting individual and community ideation through the role of convention in semiosis (activity with signs). Because the semiosis of the individual is mediated by the community through the adoption of certain ways of thinking and representing ideas as conventional, the growth of (mathematical) knowledge manifests continuity. Peirce (1992) cast further light on what he meant by continuity in his *law of mind*:

Logical analysis applied to mental phenomena shows that there is but one law of mind, namely, that ideas tend to spread continuously and to affect certain others which stand to them in a peculiar relation of affectability. In this spreading they lose intensity, and especially the power of affecting others, but gain generality and become welded with other ideas. (p. 313)

Because of the importance of personal interpretations in forging a community of thinkers with its conventions, and thus in the continuity of ideas, Peirce formulated three kinds of interpretant in his semiotic model. Accordingly, he used triads not only in his semiotic model including object, representamen (sometimes called the sign), and interpretant, but also in the types of each of these components. This model includes the need for expression or communication:

“Expression is a kind of representation or signification. A sign is a third mediating between the mind addressed and the object represented” (Peirce, 1992, p. 281). In an act of communication, then, there are three kinds of interpretant, as follows:

- the “*Intentional* Interpretant, which is a determination of the mind of the utterer”;
- the “*Effectual* Interpretant, which is a determination of the mind of the interpreter”; and
- the “*Communicational* Interpretant, or say the *Cominterpretant*, which is a determination of that mind into which the minds of utterer and interpreter have to be fused in order that any communication should take place.” (Peirce, 1998, p. 478)

It is the latter fused mind that Peirce designated the *commens*. There is a clear resonance of the commens with culture defined as a set of shared understandings (in the previous section).

For the continuity of mathematical ideas and their evolution in the history of mathematics, a central requirement is a community of thinkers who share a “fused mind” sufficiently to communicate effectively with one another-and with posterity through their artifacts-through this commens. Both the intentional and the effectual interpretants are important

for communication (and have the implicit potential for miscommunication). But the third member of this triad, the interpretant generated by the commens, leads to the adoption of conventional signs by an intellectual community.

D'Ambrosio and others following the tradition of historiography that he has inaugurated (Selin, 2000) have introduced a different commens. Future generations, through synechism, will judge the impact of this historiography on the ontology and epistemology of mathematics in general, and of mathematics education in particular.

#### Empirical Fields That Incorporate Culture in Mathematics Education

The last section dealt with some theoretical fields that have been useful, or have the potential to be useful, in research on the role of culture in mathematics education. In the current section, some associated empirical fields are discussed. Empirical fields entail broad methodologies of empirical research, but in addition to these, in this section specific methods of research and also the participants, data, and results of selected studies are introduced. These details of participants, time, and place in the research are termed the *empirical settings* (Brown & Dowling, 1998). Thus appropriate empirical fields and settings are the focus of this section, along with some results of research on the role of culture in mathematics education.

Because ethnography is the special province of holistic cultural anthropological research (Eisenhart, 1988), which has as a broad goal the understanding of various cultures, it is natural to expect that ethnography (sometimes in modified form) would be used by researchers who are interested in the role of culture in teaching and learning mathematics. This is indeed the case. However, researchers in mathematics education in general sometimes call their studies ethnography when they have not spent long enough in the field to warrant that term for their research (Eisenhart, 1988). For instance, valuable as studies such as that of Millroy (1992) are

for mathematics educators to learn more about the use of mathematical ideas in out-of-school settings such as that of Millroy's carpenters in Cape Town, South Africa, the  $5\frac{1}{2}$  month duration of her fieldwork might be considered brief for an ethnography, and most ethnographic studies in mathematics education have a shorter duration than Millroy's. For this reason, the field of such research is often more appropriately called *case studies* (Merriam, 1998), because they satisfy the criterion of being bounded in particular ways. If such studies are investigating a particular topic, such as ways of bringing out-of-school mathematics into the classroom, and using several cases to do so, they could be called *instrumental case studies* (Stake, 2000). Where the individual cases themselves are the focus, case studies are *intrinsic* (ibid.).

Most of the studies mentioned in this section may be regarded as instrumental case studies in the sense explained in the foregoing. Research that concerns in-school and out-of-school mathematical practices falls into two main categories. In the first category are studies that attempt to build bridges between informal or nonformal mathematical practices (Bishop, Mellin-Olsen, & van Dormolen, 1991) and those of formal school mathematics. In the second category are studies that link with formal mathematics education less directly but with the potential to deepen understanding of this education, by exploring the culture of mathematics in various workplaces. The studies in both of these categories are too numerous for a summary chapter such as this one to do full justice to the aims, research questions, methodologies and methods for data collection, and the subtleties unearthed in the results of such research. (Again, the reader is reminded to read the original literature for depth of coverage.) What follows is an introduction to the range of research in each of these two categories, with discussion of some of the empirical settings, a sampling of issues addressed, and some of the results.

### *Linking Mathematics Learning Out-Of-School and In-School*

In the first category, studies that specifically investigated linking out-of-school and in-school mathematics are of several types. Some researchers interviewed parents of minority learners and their teachers to investigate relationships between home and school learning of mathematics. In England, Abreu and colleagues conducted such interviews for the purpose of investigating how the parents of immigrant learners are able to support their children in their transitions to learning mathematics in a new school culture (Abreu, Cline & Shamsi, 2002). Civil and colleagues in the USA interviewed such parents for the purpose of finding out which home practices of immigrant families might be suitable for pedagogical purposes in mathematics classrooms (Civil, 1990, 1995, 2002; Civil & Andrade, 2002). Related empirical studies in which researchers investigated the transitions experienced by immigrant children in their learning of mathematics have been reported by Bishop (2002a) in Australia and by Gorgorio, Planas, and Vilella (2002) in Catalonia, Spain.

In the next type of study in this category, research was conducted in mathematics classrooms to investigate the effects of bringing out-of-school practices into that arena. Using video recording as a data collection method, Brenner (2002) investigated how four teachers and their junior high school students went about an activity in which the students were required to decide cooperatively which of two fictitious pizza companies should be given the contract to supply pizza to the school cafeteria. In a different classroom video study, Moschkovich's (2002b) research addressed the mathematical activities of seventh-grade students during an architectural design project: Students working collaboratively tried to design working and living quarters for a team of scientists in Antarctica in such a way that space would be maximized while still taking into account the heating costs of various designs. These studies emphasized the

point that through their pedagogy and instructional decisions in the classroom, teachers are an important component of the success (or lack of success) of attempts to use out-of-school practices effectively in mathematics classrooms. Beliefs of teachers about the nature of mathematics, what culture is, and its role in the classroom learning of mathematics are strong factors in teachers' decisions in this regard (Civil, 2002; Presmeg, 2002b).

Like the researchers in the preceding paragraphs, Presmeg (1997, 1998b, 2002b, in press), Hall (2000), and Adeyemi (2004) recognized the lack of congruence of out-of-school cultural practices and those "same" practices when brought into the school mathematics classroom (Walkerdine, 1988, 1990, 1997). Their research used semiotic chaining as a theoretical tool (see previous section) in an attempt to bridge the gaps, not only between these practices in- and out-of-school, but also between mathematical ideas implicit in these activities and the formal mathematics of the syllabi that teachers are expected to use in their practices. The fourth-grade teachers with whom Hall worked built chains of signifiers that they implemented with their classes, starting with a cultural practice of at least some of their students and aimed at linking mathematical ideas in this practice in a number of steps with a formal school mathematics topic. Chains constructed by the teachers were designated as either *intercultural*-bridging two or more cultures, or *intracultural*-having a chain that remained within a single culture. Examples of the first type involved number of children in students' families, pizzas, coins, and measurement of students' hands, linking in a series of steps with classroom mathematical concepts. These are intercultural because the cultures and discourse of students' homes or activities are linked with the different discourse and culture of classroom mathematics, for instance, the making of bar graphs. Manipulatives were frequently used as intermediate links in these chains. The intracultural type, involving chains that were developed within the culture of

a single activity, was evidenced in a chain involving baseball team statistics. The movement along the chain could be summarized as follows:

Baseball Game → Hits vs. At Bats → Success Fraction → Batting Average.

It was not the activity that was preserved throughout the chain, but merely the culture of baseball (at least as it was imported into the classroom practice) within which the chain was developed.

The need for interpretation at each link in the chain led to further theoretical developments using a triadic nested model that was a better lens for interpreting the results (Presmeg, in press).

One study that is ethnographic in its scope and methodology has less explicit claims to link out-of-school and in-school mathematics, although implicit ties between the two contexts are present. This study is a thorough investigation of mathematical elements in learning the practice of selling newspapers in the streets by young boys (called in Portuguese *ardinas*) on the island of Cabo Verde in the Atlantic ocean (Santos & Matos, 2002). Using Lave and Wenger's (1991) theoretical framework of legitimate peripheral participation, Santos and Matos described the goal of their research as follows.

Our goal was to look into the ways (mathematics) learning relates to forms of participation in social practice in an environment where mathematics is present but that escapes the characteristics of the school environment. Because we believe that culture is an unavoidable fact that shapes our way of seeing and analyzing things, we decided to look at a culturally distinct practice and that constituted a really strange domain for us: the practice of the *ardinas* at Cabo Verde islands in Africa. (p. 81)

In addition to interesting explicit and implicit mathematical aspects of the changing practice of the *ardinas* as they moved from being newcomers to full participants, methodological difficulties in this kind of research were foregrounded by Santos and Matos (2002):

The fact that the research is studying a phenomena [sic] which was almost totally strange to us in most of its aspects, led us to realize that we had to go through a process which should involve, to a certain extent, our participation in the (*ardinas*' ) practice with the explicit (for us and for them) goal of learning it but not in order to be a full member of that community of practice. This starting point (more in terms of knowing that there are more things that we don't know than that we know) opened that community of practice to us but also gave us consciousness that methodological issues were central in this research. (p. 120)

Many of the difficulties that Santos and Matos described with sensitivity and vividness are common to most anthropological research and thus also impact investigations of cultural practices in the field of mathematics education. They were required to become part of the culture sufficiently to be able to interpret it to others who are not participants, but at the same time not to become so immersed in the culture that it would be transparent-a lens that is not the focus of attention because one looks *through* it. They faced the difficulty that entering the group as an outsider might in fact change the culture of that group to a greater or lesser extent. On several occasions there was evidence that the mathematical reporting by key informants among the *ardinas* was influenced by the fact that the fieldworker was a Portuguese-speaking woman, whom these boys might have associated with their schoolteachers. Finally, the researchers recognized the not inconsiderable difficulties associated with practical matters concerned with collecting data in the street.

The investigation of the *ardinas*' mathematical thinking could be classified as an ethnographic workplace study. Thus this short account of this research provides a transition to the second aspect of this empirical section of the chapter, mathematics in the workplace.

*Culture of Mathematics in the Workplace*

In the second category of studies in this section, the culture of mathematics in various workplaces was investigated intensively by FitzSimons (2002), by Noss and colleagues (Noss & Hoyles, 1996a; Noss, Hoyles & Pozzi, 2000; Noss, Pozzi & Hoyles, 1999; Pozzi, Noss, and Hoyles, 1998), and by several other researchers (see later). FitzSimons investigated the culture and epistemology of mathematics in Technical and Further Education in Australia, and how this education related or failed to relate to the cultures of mathematics as used in workplaces. Noss, Hoyles, and Pozzi examined workplace mathematics in more specific detail: *Inter alia* their studies addressed mathematical aspects of banking and nursing practices. A common thread in these studies is the *demathematization* of the workplace: “As the seminal work of researchers Richard Noss and Celia Hoyles among others indicates, mathematics actually used in the workplace is contingent and rarely utilizes ‘school mathematics’ algorithms in their entirety, if at all, or necessarily correctly” (FitzSimons, 2002, p. 147).

The same point was brought out strongly, but contingently, in an investigation of mathematical activity in automobile production work in the USA (Smith, 2002). One result of Smith’s study was that “the organizational structure and management of automobile production workplaces directly influenced the level of mathematics expected of production workers” (p. 112). This level varied from a minimal expectation on assembly lines—designed to be “worker-proof”—to “a surprisingly high level of spatial and geometric competence, which outstripped the preparation that most K–12 curricula provide” (p.112), e.g., as manifested by skilled machinists who translated between two- and three-dimensional space with sophistication and accuracy in creating products that were sometimes one-of-a-kind. Organization that used “lean manufacturing principles” (p. 124) to move away from assembly lines and give more autonomy to workers was more likely to enhance and require these highly developed forms of mathematical

thinking. Workers are not usually allowed to decide what level of mathematics will be required in their work:

The fact that the organization of production systems and work practices mediates and in some cases limits workers' mathematical expectations and their access to workplace mathematics is a reminder that the everyday mathematics of work is inseparable from issues of power and authority. In large measure, someone other than the production workers themselves decides when, how often, and how deeply they will be called on to think mathematically. (Smith, 2002, p. 130)

Other sources of insights into the mathematical ideas involved in practices of various workplaces are detailed in FitzSimons's (2002) book. These include the following:

operators in the light metals industry (Buckingham, 1997); front-desk motel and airline staff (Kanes, 1997a, 1997b); landless peasants in Brazil ... (Knijnik, 1996, 1997, 1998); carpet layers (Masingila, 1993); commercial pilots (Noss, Hoyles, & Pozzi, 2000); ... draughtspersons (Strässer, 1998); semi-skilled operators (Wedeg, 1998b, 2000a, 2000b) and swimming pool construction workers (Zevenbergen, 1996). Collectively, these reports highlight not only the breadth and depth of mathematical concepts encountered in the workplace, but underline the complex levels of interactions in the broad range of professional competencies as outlined above, where mathematical knowledge can come into play – when permitted by (or in spite of) management. (p. 72)

In addition to this broad range of reported research, in an early study Mary Harris (1987) investigated “women’s work,” challenging her readers to “derive a general expression for [knitting] the heel of a sock” (p. 28). The more recent mathematics education conferences have included presentations on the topic of the mathematics of the workplace (e.g., Strässer, 1998).

Strässer and Williams (2001) organized a Discussion group on “Work-Related Mathematics Education” at the 25th conference of the International Group for the Psychology of Mathematics Education. They presented the aims of this Discussion Group in the following terms.

Discussions in the group should aim at better understanding the contradictory trends of hiding or revealing workplace maths by means of artefacts and discourses, and identify problems and potentials for teaching and learning work-related mathematics. Current research into the use of mathematics at work (with the spectrum from traditional statistics to ethnomethodology) seems to favour case studies in a participatory style, while different ways of “stimulated recall” are also in use. (p. 265)

From the foregoing, the breadth of this field, the variety of practices that have been investigated from a mathematical point of view, and the scope of methodologies employed are apparent.

Many of these studies may be characterized as investigations in ethnomathematics, for example, those of Masingila (1993, 1994) with the art of carpet laying, and Harris (1987). This overlap in classification also applies to research on mathematics in the world of work that is concerned with its artifacts and tools, and with its technology. The confluence of technology and culture in mathematics education is the topic of the last part of this section.

### *Influences of Technology*

A study that stands at the intersection of workplace mathematics studies and those that are concerned with the role and influence of technology in mathematics education is that by Magajna and Monaghan (2003). In their research they investigated both the mathematical elements and the use of computer aided design (CAD) and manufacture (CAM) in the work of six skilled technicians who designed and produced moulds for glass factories. The technicians specialized in “moulds for containers of intricate shapes” (p. 102), such as bottles in the form of twisted

pyramids, stars, or guitars. The study is interesting because the mathematics used by the CAD/CAM technicians in various stages of their work (for instance, calculating the interior volume of a mould) was not elementary, and in theory school-learned mathematics could have been used. However, the researchers reported as follows.

Although the technicians did not consider their activity was related to school mathematics there is evidence that in making sense of their practice they resorted to (a form of) school mathematics. The role of technology in technicians' mathematical activity was crucial: not only were the mathematical procedures they used shaped by the technology they used but the mathematics was a means to achieve technological results. Further to this the mathematics employed by the technicians must be interpreted within the goal-oriented behaviour of workers who 'live' the imperatives and constraints of the factory's production cycle. (p. 101)

An implication of results such as these, resonating with the conclusions of other workplace studies, is that there is no direct path from school mathematics to the mathematics used, sometimes indirectly, in various occupations, where the specific practices are learned on the job. It may not be possible then to gear a mathematics curriculum, even in vocational education (FitzSimons, 2002), to the specific requirements of a number of vocations simultaneously. However, as technology has developed in the last 6 decades, its influence has been felt in mathematics curricula of various time periods, as illustrated by Kelly (2003) and described next.

Each of the mathematics curriculum movements of the last century felt the impact of the state of the current technology of that period. Building on the assertion that "the tool defines the skill," Kelly (2003, p. 1041) described the influence of technological developments on mathematics curricula in four different time periods, overlapping with the following crucial years:

- 1942-the mainframe computer;
- 1967-the first four-function calculator;
- 1978-the personal computer (microcomputer);
- 1985-the graphing calculator.

In a broad sense, then, technology of various types has influenced considerably the culture of mathematics education taken as a complex of shared understandings (Stenhouse, 1967) about the nature of mathematics and its pedagogy. However, this influence does not imply unanimity about *what* technology should be brought into classrooms, or *how* it should be used. Contestation and conflict accompanied the growing use of computers and calculators of increasing sophistication in mathematics education, as Kelly's account showed vividly. He pointed out that the influence has not only permeated the culture of mathematics education through various curricula, but that as the technology changed and became more interactive through graphical user interfaces, the potential for a different kind of learning of mathematics became possible, and hence the need arose for a different kind of instruction. However, two impediments to the incorporation of new technology into mathematics classrooms have been the inaccessibility of the technology-exacerbated particularly in developing countries (Setati, 2003b)-and a lack of teacher readiness (Kelly, 2003).

Kelly's (2003) portrayal also suggests the importance of the avenues opened up by the graphing calculator, especially in-but not limited to-secondary and post-secondary schooling. Use of graphing calculators and computers impacts the nature of mathematics learned at all levels. The research in this growing field is beyond the scope of this chapter, but two examples highlight the potential of graphing calculators and computers to change the culture of learning mathematics. First, the research of Ricardo Nemirovsky (2002) and his colleagues (Nemirovsky,

Tierney, & Wright, 1998) demonstrates how the use of motion detectors and associated technology changes the culture of learning from one of fostering *formal* generalizations in mathematics (e.g., “all  $x$  are  $y$ ”), to one of constructing *situated* generalizations. The latter kind of generalization is “embedded in how people relate to and participate in tasks, events, and conversations” (Nemirovsky, 2002, p. 250)-and it is “loaded” with the values of “grasping the circumstances and transforming aspects that appear to be just ordinary or incidental into objects of reflection and significance” (p. 251). Nemirovsky illustrated this kind of generalization in the informal mathematical constructions of Clio, an 11-year-old girl working with the problem of trying to predict, based on the graph created on a computer screen by a motion detector, where a toy train is situated in a tunnel.

A second example is from the doctoral research of Paul Yu (2004), who investigated middle grades students’ learning of geometry via the interactive geometric program called Shape Makers (Battista, 1998). Yu’s research demonstrated how use of an interactive computer program such as this one has the potential to change the order of acquisition of the van Hiele (1986) levels of learning geometry. For instance, for these learners, a trapezoid was *what the trapezoid maker makes* in comparison with other shapes, and only later would they focus on individual appearance and properties of particular shapes.

The influence of dynamic geometry systems has been considerable, and research on the impact of such interactive software on the culture of school learning of geometry is important and ongoing. As Kelly (2003) pointed out, not only does the tool define the skill, but the development of new tools changes the culture and practices of mathematics itself, for instance in the impact of computer algebra systems on mathematics, or in changes made possible by the ease

with which the computer can perform large numbers of routines that are beyond human capabilities.

In summary, then, technology, by entering all avenues of life, influences not only the mathematics of the curriculum and the ontology of mathematics itself, but also the culture of the future, through its children.

This section has examined in broad detail some of the empirical research relating to the role of culture in teaching and learning mathematics. Aspects that were considered included the linking of out-of-school and in-school mathematics learning, both from the point of view of comparing these different cultural practices and from the point of view of building bridges between cultural practices and the classroom learning of mathematics-of “bringing in the world” (Zaslavsky, 1996). A related strand that was outlined was the culture of mathematics in the workplace, and finally, some aspects of the cultural influence of technology on the learning of mathematics were suggested. Clearly in all of these strands ongoing research is needed and in progress. However, perhaps just as significantly, research is needed that will increase understanding of and highlight how these strands are related amongst themselves. For instance, as the use of technology both in the workplace and in the mathematics classroom changes the culture of learning mathematics in these broad arenas, hints of avenues to be explored in future work appear, for example in whether and how technology has the potential to bridge or decrease the gap between formal and informal mathematical knowledge. The theme of technology and culture in future research is elaborated, along with other significant areas, in the final section.

#### Future Directions for Research on Culture in Mathematics Education

The final section uses a wider lens to zoom out and consider areas of research on culture in mathematics education that require development or are likely to assume increasing importance

in the years ahead. By its nature, this section is speculative, but trends are already apparent that are likely to continue. The influence of technology is one such trend (Morgan, 1994; Noss, 1994).

### *Technology*

Already in 1988, Noss pointed out the cultural entailments of the computer in mathematics education, as follows: “Making sense of the advent of the computer into the mathematics classroom entails a cultural perspective, not least because of the ways in which children are developing the computer culture by appropriating the technology for their own ends” (p. 251). He elaborated, “The key point is that children see computer screens as ‘theirs’, as a part of a predominantly adult culture which they can appropriate and use for their own ends” (p. 257). As these children become adults with a facility with technology beyond that of their parents (Margaret Mead’s *prefigurative enculturation* comes into play as children *teach* their parents), the culture of mathematics education in all its aspects is likely to change in fundamental ways. Noss (1988) and Noss and Hoyles (1996b) put forward a vision for the role that computer microworlds have played and might play in the future of mathematics education. With accelerating changes in platforms, designs, and software, research must keep pace and inform resulting changes in the cultures of teaching and learning of mathematics in schools. An aspect of potential use for the computer stems from its ability to mimic reality in a world of virtual reality. Noting the complex relationship between formal and informal mathematics (ethnomathematics), Noss (1988) suggested, “I propose that the technology itself – specifically the computer – can be the instrument for bridging the gap between the two” (p. 252). This area remains one in which research is needed, both in its own right and in ways that the change of

context of bringing the virtual reality of computer images into mathematics classrooms changes the culture of teaching and learning in those classrooms.

*Bridging Mathematics In-School and Out-Of-School*

Resonating with the Realistic Mathematics Education viewpoint that *real* contexts are not confined to those concrete situations with which learners are familiar (van den Heuvel-Panhuizen, 2003), Carraher and Schliemann (2002) made a useful distinction between *realism* and *meaningfulness*:

What makes everyday mathematics powerful is not the concreteness of the objects or the everyday realism of the situations, but the meaning attached to the problems under consideration (Schliemann, 1995). In addition meaningfulness must be distinguished from realism (D. W. Carraher & Schliemann, 1991). It is true that engaging in everyday activities such as buying and selling, sharing, or betting may help students establish links between their experience and intuitions already acquired and topics to be learned in school. However, we believe it would be a fundamental mistake that schools attempt to emulate out-of-school institutions. After all, the goals and purposes of schools are not the same as those of other institutions. (p. 137)

The dilemma, then, is *how* to incorporate out-of-school practices in school mathematics classrooms in ways that are meaningful to students and that do not trivialize the mathematical ideas inherent in those practices. This issue remains a significant one for mathematics education research.

Reported research (Civil, 2002; Masingila, 2002; Presmeg, 2002b) has shown that learners' beliefs about the nature of mathematics affect what mathematics they identify *as* mathematics in out-of-school settings. Students' and teachers' conceptions of mathematics as

decontextualized and abstract may limit what can be accomplished in terms of meaningfulness derived from out-of-school practices (Presmeg, 2002b). And yet, abstract thinking is not necessarily antagonistic to the idea of reasoning in particular contexts, as Carraher and Schliemann (2002) pointed out, which led them to propose the construct of *situated generalization*. They summarized these issues as follows.

Research sorely needs to find theoretical room for contexts that are not reducible to physical settings or social structures to which the student is passively subjected. Contexts can be imagined, alluded to, insinuated, explicitly created on the fly, or carefully constructed over long periods of time by teachers and students. Much of the work in developing flexible mathematical knowledge depends on our ability to recontextualize problems – to see them from diverse and fresh points of view and to draw upon our former experience, including formal mathematical learning. Mathematization is not to be opposed to contextualization, since it always involves thinking in contexts. Even the apparently context-free activity of applying syntax transformation rules to algebraic expressions can involve meaningful contexts, particularly for experienced mathematicians. (p. 147)

They mentioned the irony that the mechanical following of algorithms characterizes the approaches of both highly unsuccessful and highly successful mathematical thinkers.

Taking into account both the need for meaningful contexts in the learning of mathematics and the necessity of developing mathematical ideas in the direction of abstraction and generalization (in the flexible sense, not to be confused with decontextualization), at least two extant fields of research have the potential to address these issues in significant ways. The notions of *horizontal* and *vertical* mathematizing that have informed Realistic Mathematics Education research for several decades (Treffers, 1993) could resolve a seeming conflict between

abstraction and context. In harmony with these ideas, recent attempts to use semiotic theories in linking out-of-school and in-school mathematics also have the potential for further development (Hall & Presmeg, 2000; Presmeg, in press; Yackel, Stephan, Rasmussen, & Underwood, 2003).

As Moschkovich (1995) pointed out, a tension exists between educators' attempts to engage learners in "real world" mathematics in classrooms and movements to make mathematics classrooms reflect the practices of mathematicians. Multicultural mathematics materials for use in classrooms have been available for some time (Krause, 1983; Zaslavsky, 1991, 1996). However, the tensions, the contradictions, and the complexity of trying to incorporate practices for which "making change" serves as a metonymy at the same time that students are "making mathematics" (Moschkovich, 1995) will engage researchers in mathematics education for some time to come. Bibliographies such as that compiled by Wilson and Mosquera (1991) will continue to be necessary, to inform both researchers and practitioners what has already been accomplished as the field of culture in mathematics education continues to grow.

### *Teacher Education*

In all of the foregoing areas of potential cultural research in mathematics education, the role of the teacher remains important. Noting that concrete and abstract domains in mathematical thinking are not necessarily disparate, Noss (1988) suggested,

The key idea is that of focusing attention on the important relationships involved, a role in which – as Weir (1987) points out – the computer is rather well cast; but not without the conscious intervention of educators, and the careful development of an ambient learning culture. (p. 263)

Bishop (1988a, 1998b) was also intensely aware of the role of the *mathematical enculturators* in personifying the values that are inherent in the teaching and learning of mathematics. In the final

chapter of his seminal book (1988a) he suggested requirements for the education-rather than the more restricted notion of “training”-of those who will be mathematics teachers, at both the elementary and secondary levels. He did not distinguish between these levels, for teachers at both elementary school and secondary school have important mathematical enculturation roles. Bishop (1988a) summarized the necessary criteria as follows.

I propose, then, these four criteria for the selection of suitable Mathematical enculturators:

- ability to ‘personify’ the mathematical culture
- commitment to the Mathematics enculturation process
- ability to communicate Mathematical ideas and values
- acceptance of accountability to the Mathematical cultural group. (p. 168)

These ideas still seem timely; in fact the literature on discourse and communication has broadened in the decades since these words were written, to suggest that communication amongst all involved in the negotiation of the cultures of mathematics classrooms (teacher and learners) plays a significant role in the learning of mathematics in those arenas (Cobb et al., 1997; Dörfler, 2000; Sfard, 2000). *Language and Communication in Mathematics Education* was the title of a Topic Study Group at the 10th International Congress on Mathematical Education (Copenhagen, 2004), and the literature in this field is already extensive. But *how* to educate future teachers of mathematics to satisfy Bishop’s four criteria is still an open field of research.

As Arcavi (2002) acknowledged, much has already been accomplished in curriculum development, research on teachers’ beliefs and practices, and “the development of a classroom culture that functions in ways inspired by everyday practices of academic mathematics” (p. 27). However, open questions still exist concerning ways of using the recognition that the transition from out-of-school mathematical practices to those within school is sometimes not

straightforward, in order to inform the practices of mathematics teaching. Arcavi (2002) gave examples of such questions.

However, much remains to be researched. For example, is it always possible to smooth the transition between familiar and everyday contexts, in which students use ad hoc strategies to solve problems, and academic contexts in which more general, formal, and decontextualized mathematics is to be learned? Are there breaking points? If so, what is their nature? Studies in everyday mathematics and in ethnomathematics are very important contributions, not only because of their inherent value but also because of the reflection they provoke in the mathematics education community at large. There is much to be gained from those contributions. (p. 28)

Clearly ethnomathematics conceived as a research program (D'Ambrosio, 2000) has already permeated the cultures of mathematics education research and practice in various ways. The influence, and the need for research that addresses the complexities of the issues involved, are ongoing.

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Useful web sites on culture and the learning and teaching of mathematics:

<http://www.csus.edu/indiv/o/oreyd/once/once.htm>

[http://www.geometry.net/pure\\_and\\_applied\\_math/ethnomathematics.html](http://www.geometry.net/pure_and_applied_math/ethnomathematics.html)

<http://www.dm.unipi.it/~jama/ethno/>

<http://phoenix.sce.fct.unl.pt/GEPEm/>

<http://www.fe.unb.br/etnomatematica/>

<http://www2.fe.usp.br/~etnomat>

<http://web.nmsu.edu/~pscott/spanish.htm>

<http://etnomatematica.univalle.edu.co>

<http://www.rpi.edu/~eglash/isgem.htm>

<http://chronicle.com/colloquy/2000/ethnomath/ethnomath.htm>

<http://chronicle.com/free/v47/i06/06a01601.htm>

<http://chronicle.com/colloquy/2000/ethnomath/re.htm>

<http://www.ecsu.ctstateu.edu/depts/edu/projects/ethnomath.html>