Engaging Students' Minds by Bringing Trigonometry to Life

Stephen J. Hegedus, Sara K. Dalton
SimCalc Projects
University of Massachusetts Dartmouth
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Aims:

- Work with explorations of circular functions in SimCalc MathWorlds
- Motion around a unit circle links to periodic motion; an understanding of "2π" and its "measurement" of a unit circle; how sin varies with respect to y; how cos varies with respect to x; both as 1-D distance measurements (i.e., periodic motion of points along the x-axis and then along the y-axis) and 2-D co-variation

Extended:

- What are our parameters of variation, i.e. y=a*sin*(bx+c)+d?
- How does movement around a circle help us understand trigonometric functions?
- How do we fuse our geometric explorations with triangles and circles with our present activities with physical motion, animation of motion, and understanding of how our "mathematical motion" enables us to make sense of sinusoidal variation?

Activity 1: Introduction to SimCalc MathWorlds

Introduce the basic features of SimCalc MathWorlds
Dynamic, interactive graphs
Algebraic forms
Re-animating motion from imported data
Discuss introduction to MW:
Walking the line—how can we re-create y=2x?

Activity 2: Introduction to importing motion using CBR

Walk a circle in 6 seconds of diameter equal to 6 feet.

Ask for a volunteer. Ask class what they expect to see by drawing on whiteboard in advance

What graph do we expect to see?

How does a circular motion relate to our graph and our simulated motion?

Questions:

- How long did it take to walk the circle?
- Is that represented in the graph?
- Is that represented in the motion?
- How far do I travel?
- How fast do I travel? How can I represent that graphically?
- Now discuss what they see with respect to the graph.
- Compare differences. Discuss. What does the motion tell us?
- If we change starting positions what happens?

Activity 3: Amplitude

All start near the board.

Several people walk a circle whose diameter is equal to 3 times their walk-up number (1, 2, 3, i.e., 3, 6, 9}

Questions:
What do you expect to see in terms of relative motions vs. what you saw your classmates do?

What graphs do you expect to see?
How do they differ qualitatively/quantitatively?

What graph have I tried to model in my motion?

Can we describe it in terms of y=Asin(Bx)?

Activity 4: Period
If I walk in 6 seconds what is that approximately?

Questions:
What is pi?
What is 2pi as a quantity in time?
If I use a unit circle – how far do I travel around the circle?
If I travel around the circle once how far do I travel relative to where I start?
How does this relate to the graph I just created?
How is it relative to my own/my classmate's motion?
What is the period of my motion?

Working Definition: I walk one "thing" in a certain amount of time, e.g., a 400m track, a street block, etc. If I walk back, or walk the track again at the same speed I will ***repeat*** my motion.

My motion is repetitive. I complete the motion. One period passes. A period is unit of measurement like one class = 1hr 15min.

If I repeat it—the same block of time goes by and the same distance is covered.

Repeating a period (a block) means it is **periodic**—repetitive.

At the heart of Periodicity is the Mathematics of Change and Variation

Statement: Circular motion is Periodic—is this observable?

What is a period?
Describe in terms of physical motion and quantify.

If I revolved the circle in “2pi” seconds what is my period? Estimate 2pi and relate to 6 seconds.

What graph have I tried to model in my motion?

Can we describe it in terms of y=Asin(Bx-C)?

What is B? What is a good estimate?

When our period was 6 seconds or roughly 2pi—what was our sine function that best modeled our walk?

Try y=Asinx

What does that tell us about the coefficient of x?

Someone try 12 seconds to walk around circle (same).

What do you expect to observe (motion and graph)?

What do you observe in terms of the motion (did you feel yourself moving more slowly)? Interpret that in terms of revolution? In terms of period.

Can you feel periodicity?

Answers:
If one period = 2pi seconds : observe y=sinx
And then one period = 4pi seconds: observe y = sin(x/2)
What model best fits?: i.e., b=1 and then b=1/2
Where does 1 and _ show up in the model?

B=2pi/2pi=1 for 2pi revolution
B=2pi/4pi=1/2 for 4pi revolution

What do you think the formula is for B?
Test the formula with other revolutions.

Now we might test with best fit graphs first or see if people can see a relationship between period of revolution and B first.

Activity 5: Qualifying horizontal and vertical motion in terms of sin and cosine

Use both horizontal and vertical worlds. Now we need to carefully examine the whole motion.

Activity 6: Explore D
Start at various distances from the wall but call for similar motions

Compare and discuss

Activity 7: Explore C
Now start at different positions around the circle:

Circle of diameter 9 feet—talk about measurement. Walk around in 6 seconds

Walking anti-clockwise leads to pulling the graph to the LEFT—a dragging/shifting motion but amplitude and periodicity is invariant BECAUSE group number and time of revolution is unchanged

Discuss walking clockwise.

With the circle to the wall what graphs are generated when our motions start at 6, 9, 12 and 3 o'clock?

Extension—Activity 8: Saw-Tooth functions

Build a PW function that is similar to the sinx function—what does it look like?
Now ask each group to build one relative to its function (i.e., based upon its group number).

What is similar about the graphs?
How do the motions relate?
What would be a better approximation?
Can we construct a better PW function? How?
Describe your reasoning/construction/thinking.