Representation & Communication

Keywords: Digital semiotics, co-action, mathematical expressivity, dynamic mathematics; Meaningful participation in SimCalc environments

Stephen Hegedus
Professor, Department of Science, Technology, Engineering and Mathematics
Director, James J Kaput Center for Research & Innovation in Mathematics Education
Agenda

• What is the problem in learning mathematics?
• The issue of motivation
• The role of technology - we need to step back 35,000 years
• The new infrastructures - representation and communication - what is the new mathematics education?
Big Problems

- Algebra Problem (RAND Report 2002)
- Student motivation and alienation in the nation’s schools, especially urban secondary schools (National Research Council, 2003)
- Widely acknowledged unfulfilled promise of technology in education, especially mathematics education (e.g., Cuban, 2001)
Exhibit ES-1. Percentage of Students With Access to Computers for Mathematics Instruction in 2004–05

Exhibit reads: Fifty-five percent of fourth-graders and 44 percent of eighth-graders had access to computers in mathematics classrooms. Seventy-four percent of fourth-graders and 79 percent of eighth-graders had access to computers for mathematics in school computer labs or media centers. Thirty percent of fourth-graders and 36 percent of eighth-graders did not use computers in mathematics. Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.
Exhibit ES-3. Percentage of Students Whose Teachers Used Computers in Mathematics Instruction at Least Once a Week in 2004–05

Exhibit reads: The teachers of 9 percent of fourth-graders and 13 percent of eighth-graders used computers at least once a week to present mathematics concepts. The teachers of 11 percent of fourth-graders and 32 percent of eighth-graders used computers at least once a week to post homework, assignment, or schedule information on the Web. Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.
Motivation

• Extrinsic motivation such as rewards can have an undermining effect and decrease intrinsic motivation

• Intrinsic motivation reflects the propensity for humans to engage in activities that interest them

• Key factors for participation but have been orthogonal to the field of inquiry to the development and instruction of content

• Motivational strategies have been in the form of incentivizing students because it is fun
A new approach

- Students can be motivated because they want to participate more fully in what their classroom is doing now
- Link motivation and mathematics through participation
Some historical perspectives

- The history of writing, from pictographs to alphabets, alphabets to the printing press, teaches: How the presence of a new technology transforms the practice and redefines the nature of that practice...
We are in the middle of a transition that has its deep origins around 35,000 years ago and is still gaining momentum.

We might describe this transition as based upon:

- External systems of representations (semiotic systems),
- Communication, and
- Cognitive and cultural mediators (poetry, music, computers...)
Digital Semiotic: A vision of the Dialogical medium

The development of portability of information and knowledge

Clay
↓
Papyrus
↓
Paper
↓
Screen

Beginning of the history of malleability of media as part of man’s efforts to evolve communication acts
From Static to Dynamic Mathematics

What happens with mathematics when it is transcribed/embedded in a digital media?

Is its mode of existence the same?

Are new mathematical phenomena waiting to be represented?

This transcription cannot simply be described as one whose end result is:

to provide the thinker, the teacher, the student, with a more convenient amplifying device.

Technology should not serve as a prosthetic device to prop up old practices but transform the educational landscape
Attributes:

- Co-Action
- Plasticity and elasticity
- Applied epistemology
Shifting from Static to Dynamic inscriptions and the Identification of Symbolization

Stage 1. Static Inert
- Cuneiform art/clay/print outs
- Inscriptions are hardened” or “fused” with the media it is presented upon or within

Stage 2. Static Kinesthetic/Aesthetic
- Pens, pencils, erasers/Advance of color
- Kinesthetic inscriptions
Stage 3. Static Computational

- Graphing calculators
- Presentations (e.g., graphs) are artifacts of a computational response to a human’s action
- The intentional acts of a human are computationally refined.
Stage 4. Discrete Dynamic

- Spreadsheets: Spinners/Sliders in Excel
- Representations of a co-action between user and environment - a process of presentation and examination
Stage 5. Continuous Dynamic

- Avitzur’s Graphing Calculator
- Continuous navigational input
- Environment is malleable, re-animates notations and expressions on direct inputs

Math expression: $z = x^2 - y^3$
Stage 6. Continuous Dynamic Kinesthetic

- Includes kinesthetic input or co-action, to make sense of physical force, or gestural interaction through space and time
- Kinesthetic interaction (via a simple mouse pointer or even a stylus), is computationally digitized within the medium of presentation, action, and in some cases, user-intentionality.
What is a graphical representation - semiotics talks about “points”, digital semiotics talks about executable reference to see through the graphical sign.

Dynamic mathematics can “see through” graphical signs.

Graphical interpretation (symbolizing) is understanding change and motion.
Example 1: Rate

- SimCalc MathWorlds(R) a new dynamic representational infrastructure using the computational medium to extend inherited numerical, graphical and algebraic systems

- Hot links graphs to animations, Visually editable graphs, links between rates and accumulation graphs, importing physical motion
• Analogous to the historical encapsulation of structure into a notation system - Base 10 Placeholder system embodies in compact form the hierarchical exponential structure that democratized access to computation with arbitrarily large numbers.

• SimCalc RI emphasizes flexibility in yielding large combinations of curriculum, activity structures, and pedagogical strategies. Lays the foundations for Calculus.

• Additional ingredient of Connectivity both wireless and wired.
SimCalc Demo
Example 2: Families of Functions

- Focus on the intensity of interaction
- Multiple student-student interactions
- Mathematical argumentation
- Role of gesture and Pointing
- Motivated to participate as they (the students) are part of the public display
Mathematical Activity:
Create a motion for your actor that moves at a speed equal to twice your group number for 6 seconds

\[ y = 2g + 0 \]

where \( g \) is Group Number

Analysis
Students describe what the graphical representation will look by drawing each function in the air. Another student uses his fingers to gesturally describe the graph of the functions. Another holds pencils and pens in a way that describes what she thinks the family of functions will look like BEFORE they are shown.

Categorization
- Public
- Mathematical: Slope-as-Rate
- Static
- Iconic {McNeill}
What have we done?

1. Software & Curriculum are inextricably linked

2. Students participation are mathematically meaningful

3. Agency is distributed

4. Feedback is collaborative and non-verbal - enhancing metacognition

5. Mathematical structure as emergent phenomenon - students build reasoning and abstraction gradually

6. Co-action - guiding and being guided by the environment
Representational Infrastructure

Communication Infrastructure

Gesture/Deixis
Indexical

Representational Expressivity

Intentional acts
Reflections

- Representational Infrastructure and communication infrastructure transforming the educational landscape of the 21st century

- From local cognitive acts to public, social acts of expression both gesturally and metaphorically lead to informal mathematical registers
Co-Action

• It is the executability of the semiotic representations that acts as the substrate to this very complex and long process

Duval (2006) has very aptly written with respect to mathematical learning:

…there is a basic difference between mathematics and other domains of scientific knowledge. Mathematical objects, in contrast to phenomena of astronomy, physics…etc., are never accessible by perception or by instruments (microscopes, telescopes, measurements apparatus). The only way to have access to them and deal with them is using signs and semiotic representations…[for] how can [the learners] distinguish the represented object from the semiotic representation used if they cannot get access to the mathematical object apart from the semiotic representation? (p. 107)
But how then can we answer Duval’s deep question?

- As we have shown with our examples, the executability of the semiotic representations provided by the medium generates a dynamic experience that goes through a labeling-revealing, generation of field reference process driven by intentions of activity and intentionality of the user.

- What we are suggesting is the adoption and adaptation of a cognitive strategy based on the transforming power of the executability of semiotic representations as they live in the domains of abstraction provided by dynamic environments.
Attributes:

- Co-Action
- Plasticity and elasticity
- Intentionality
- Executability
- Crystallization
- Applied epistemology
Summary

- Looking “through signs” can lead to symbolization.
- Symbolic thinking occurs with {co-action}
- Co-action can occur in various social contexts
- Co-action always occurs in digital/dynamic technological environments
- Hence, focus on digital environments to enable symbolic thinking in educational environments
- To do this we need a digital semiotic theory to understand the flow of action that does this
This bar graph illustrates the mean gains from pre to post for the Comparison vs. SimCalc groups in the schools who participated in the study. In the SimCalc group, mean gain is about 2 points out of a total of 26 points. In the Comparison group, mean gain is about 1 point. This group difference is statistically significant, $t=2.465$ ($p<0.015$).
This bar chart shows the mean gain (Post-Test Score minus Pre-Test Score) for each teacher, Comparison and SimCalc, who participated in the study. The x-axis is the teacher and the y-axis is gain. The scale on the y-axis illustrates the number of points (out of a possible 26) the class gained from pre to post.
Profile for Erin

<table>
<thead>
<tr>
<th></th>
<th>Pre</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class Mean</td>
<td>0.388</td>
<td>0.431</td>
</tr>
<tr>
<td>Erin’s Mean</td>
<td>0.231</td>
<td>0.539</td>
</tr>
<tr>
<td>Class Reality Mean</td>
<td>0.766</td>
<td>0.782</td>
</tr>
<tr>
<td>Erin’s Reality Score</td>
<td>0.497</td>
<td>0.720</td>
</tr>
</tbody>
</table>

**Attitude Survey Results**

0 is strongly disagree
1 is disagree
2 is neutral
3 is agree
4 is strongly agree

<table>
<thead>
<tr>
<th>Statements</th>
<th>Pre</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>I do not like school.</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>I like math.</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>When I see a math problem, I am nervous.</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>I sometimes feel nervous talking out-loud in front of my classmates.</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>I enjoy using a computer when learning mathematics.</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>I am not comfortable using technology in math class.</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>
Students describe what the graphical representation will look by drawing each function in the air. Another student uses his fingers to gesturally describe the graph of the functions. Another holds pencils and pens in a way that describes what she thinks the family of functions will look like BEFORE they are shown.
Thoughts

• Blend of intentionality of activity and pedagogy leads to a co-action
• The environment is elastic to allow exploration
• To evolve we need an adequate ecology
• Species evolves the environment & the attributes of the ecology to make it plastic (which leads to)
• Not an educational strategy but an evolutionary theory
• “Digital semiotic” emerges out of this phenomenon
• It is not based in the epistemology of old/traditional mathematics as people use dynamic math software to explore old ideas
Reflections

• Important math for more important folk
• We owe it to our children to let them understand the principles of change and complexity at school
• We are TRANSFORMING the classroom NOT changing/reinventing it. We do this by modifying the structure of the classroom
• NEW HUMAN INFRASTRUCTURE that erodes past curriculum by opening a window to new ways of conceiving math at school
• Customizability — critical - its simple to use
• Unleashed new forms of expressivity that have been dormant in mathematics classrooms for years.
• Restructuring communication through wireless connectivity; new forms of interaction, participation and expression
• Transforming the communicative heart & infrastructure of the classroom
Next Steps

- Efficacy Work (Cluster-randomized trials in MA)
- Understanding new forms of participation, supporting activity structures, and necessary professional development
- Next steps - how can we integrate this approach in new countries and cultures?
shegedus@umassd.edu
kaputcenter.umassd.edu