Psychology and Mathematics Education

Part 1: From 1880-2005

Introduction

• From ‘and’ to ‘of’?
• Historical overview of psychological theory this past century
• Behaviourism - Cognition - Metacognition as examples of theory
• Models adhered to by mathematics education (politics, field of mathematics, society)
• Development this decade - the role of the mind in society
• Understanding more about how people learn at various stages, helps shape curriculum and inform pedagogy
• Guidelines and rich, literature sources in NCTM Standards - review

Historical overview

• Descartes
• William Wundt – methods pre-1900: introspection and experimentalism
• William James - Thorndike with Associativism (i.e. behaviourism)
  • Predominant theory for much of the century. Early principles of stimulus-response psychology, which affected pedagogy
• Gestaltism (1940’s) - introspectionism
  • Repetition does not lead to understanding
  • Rich mental structure
  • Gestalt: any segregated whole
• Kohler
• Mathematical Problem-solving - Dunker & Wertheimer - "productive thinking"
  • example: Box of Tacks, Matches and a Candle. Attach the candle to the wall
  • Parallelogram
  • Led to deeper insights into the methods of discovery via browsing and acknowledging structure

Changes in thought (mid 1950’s)

• Changing views of science
• Theories based on generalizations can never be logically true: falsification rather than verification

• Irrationalism - eg. Popper

• Lakatos - proofs and refutations – rational reconstruction of the Euler Formula V – E + F =2

• Piaget

• Vygotsky

Cognitive Psychology

• Information processing model

• Distinction between Cog Sci/Cog Psy/Cog Neuro-Psy

• Newell & Simon (1970's), AI, Search, e.g. Towers of Hanoi

• Unifying Psychology and Mathematics Education

• Resnick & Ford (1981) - Psychology of mathematics for instruction

• Davis, R (1985) - representations

• Vergnaud (1990) - mathematics and cognition

• Bishop (1980) – visualisation

• Kaput (1994; 2000) Representational Infrastructure; multiple representations

• Hegedus, Davis, Tall, Eisenberg (2000-5) Symbolic cognition – see www.symcog.org


To Mathematics Education - from models to models

• Concerns (to keep in mind):

• How do these theories help us understand more about how people think mathematically (psychology?)

• Benefits to education?

• Mathematical Psychology just about Problem-solving??

• Resources, Affect and beliefs, meta-thinking, representations, technology

• Popularising Psychology & Neuroscience – the decade of the “mathematical brain”, e.g. Butterworth, Dehaene
Part 2: Examples

A. Concept Image/Concept Definition

Work of David Tall and Shlomo Vinner

CI: Total Cognitive Structure
- mental pictures
- associated processes
- properties
- collection of experiences, e.g. subtraction always reduces an answer

Problems occur later, i.e. cognitive conflicts

CD: Form of words used to specify that concept, either rote or formal
- Attributes included
- Could be seeds for future conflict

Analysis of such and highlight Approaches to Teaching

DEDUCTIVE: \( CD \rightarrow CI \)

INDUCTIVE: \( CI \rightarrow CD \)

Examples:
  - Functions
  - Definition of Limit
  - Continuity
B. Generic Abstraction

- Extending properties of a concept to general levels and alternative contexts
- Recall of inherent properties of a concept

This can have positive and negative affects in application

Consider Tangents: 3 examples used with Sophomores

Problem of using semi-formal definitions in teaching

Generic Tangent as one abstracted as being common to a whole class of previous experiences, i.e. the concept of tangency

Discussion