**Classroom Activity 1: The Idea of Average Velocity—As Seen from a Velocity Point of View**

The Average Velocity of an object moving with *variable* Velocity is a *constant* Velocity that satisfies a special pair of conditions—covering the *same Distance* in the *same amount of Time* as the object traveling with *variable* Velocity. Our graphical approach with Step-Wise Varying Velocities makes it easier to see how the variation across the Average balances out—the Total Area above the Average Velocity equals the Total Area below the Average.

![Graphical representation of the idea](image)

Our graphical approach with Step-Wise Varying Rates makes it easier to see how the variation across the Average balances out—the *Total Area* above the *Average* equals the *Total Area* below the *Average*.

This brings several sources of intuition about Areas and Averages into play.

Our goal is to engage students in modeling the following, somewhat silly, situation that allows us to approach the idea of Average Velocity in two stages—first with a same Distance constraint and then with a same Time constraint. We will also develop the idea of Average from the two points of view—Velocity and then Position, where we make the development of the Position version into a Student Activity. The idea is that students can build off of their knowledge developed in the Velocity discussion and have a good chance to apply their knowledge of the Velocity-Position Connection to extend the Velocity way of dealing with Average Velocity to Position. Car A has been given a Step-Wise Varying Velocity and the job will be to adjust B’s Velocity appropriately to fit the story.

**Storyline for the Students:** Arturo and Bruno run a pizza place and share an apartment down the road—both of which are conveniently located on a busy highway. Arturo delivers the pizza because Bruno can’t read a map, and Bruno makes the pizza because Arturo burns the pot when he tries to boil water. It’s the end of the day and Arturo, using the top car (A), will be making 2 final pizza deliveries to regular customers (a convenience store...
and a gas station), along the highway on his way home. The plan is that he will then go home and meet Bruno to watch the third game of the play-offs. After cooking the pizzas and locking up, Arturo and Bruno head off at the same time, in separate cars.

**Part I—Same Distance, But Early:** Bruno—in Car B (the bottom car)—wants to get home in time to turn on the TV and get the snacks ready, so he travels at a constant rate. First, construct a constant Velocity function for Bruno’s car (B) that starts at the same time as Arturo, and gets him home (the same Distance), but a bit early. Ask students for suggestions first.

**Part II—Same Distance, But Late:** Now, revise Bruno’s constant Velocity trip home so he arrives a bit late to make the necessary preparations before game time. Again, ask students for suggestions first.

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**By simply extending the Time, you will overshoot the destination, so the Velocity must be decreased slightly. To stimulate discussion regarding what happens, it is worth trying the Time-extension approach first and, asking for predictions before finally running it. The key idea to bring out through discussion is that, in order for the Distance Traveled to be the same, the Area under the Step-Wise Varying Velocity graph and the constant Velocity graph must be the same.**

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**Part III—Same Distance, Same Time:** Finally, revise Bruno’s constant Velocity so that Arturo and Bruno arrive home at exactly the same time. You may want to verify this by writing the Distance Traveled for both as a sum of products of Velocity times Time.

**Areas of Regions Above and Below the Average Velocity Graph:** Ask the students: What is the Total Area of Car A’s (Arturo’s) Velocity graph above B’s (Bruno’s) Velocity graph? And, how much Area is below? They should notice how A’s variations above and below B’s constant Velocity graph balance out—the Area above equals the Area below. In particular, the maximum Distance between A and B relates to the Area above B’s graph but below A’s graph. And, they are side-by-side at any Time when the Areas above and below balance out. (You can Step through the motions to look at the motion in more detail.)

You might also want to repeat Part III for new versions of Car A’s variable Velocity trip. Be sure to use Total Distances and Times that are evenly divisible, unless you want to get into the issue of divisibility and turning off Snap-to-Grid.