Piaget believed students intellect developed through three age-related stages. Progress from non-reflective, illogical, unsystematic thinking to empirical, to finally, logical-deductive.

Van Hiele – again progress through stages but it is linked to geometric thinking, is developed through many stages and is influenced by school curriculum

Van Hiele Levels of Geometric Thought (as described by Michael Battista and Doug Clements in “Geometry and Proof,” Mathematics Teacher, January 1995)

Level 1—Visual
Level 2—Descriptive/Analytic
Level 3—Abstract/Relational
Level 4—Formal Deduction
Level 5—Rigor/Metamathematical
Level 1 – Visual
Reasoning is based on shape and appearance. Visual prompts or prototypes, e.g. it is a rectangle because it looks like a door

Level 2- Descriptive/Analytic
Reason experimentally. Identify shapes through its properties, use measure, and drawing to make meaning

Level 3 – Abstract/Analytic
Can form abstract definitions of shapes. Can categorize shapes into hierarchies and contrast with informal arguments, e.g. square is a rhombus with extra properties
Level 4 – Formal deduction
Formal reasoning by interpreting axioms, definitions and theorems

Level 5 – Rigor/Metamathematical
Students reason formally with systems. They manipulate and understand the consequences of manipulating axioms and definitions.

Deductive reasoning begins at level 3

70% of high school students begin at level 1

How can dynamic geometry environments help?