Tour 3: Algebra Potpourri

Don’t let the name of the program—The Geometer’s Sketchpad—fool you. Sketchpad has a host of tools for exploring algebra, trigonometry, and calculus, both symbolically (with equations) and graphically. In this tour, you’ll sample several of Sketchpad’s algebra features.

What You Will Learn

- How to create an $x$-$y$ coordinate system and measure the coordinates of a point
- How to define and plot functions
- How to plot two measurements as an $(x, y)$ point in the plane
- How to construct a locus

A Simple Plot in the Coordinate (x-y) Plane

**Presenter:** Allow 10 minutes for the first two sections.

At the heart of “visual” algebra is the $x$-$y$ (or Cartesian, or coordinate) plane. In this section, you’ll define a coordinate plane, measure a point’s coordinates on it, and plot a simple function.

1. Open a new sketch and choose **Define Coordinate System** from the Graph menu.
2. Using the **Point** tool, create a point somewhere other than on an axis.
3. With the point still selected, choose **Coordinates** from the Measure menu. Drag the point to see how the coordinates change.
4. Now that there’s a coordinate system, let’s plot a simple equation on it: $y = x$.
5. Choose **Plot New Function** from the Graph menu.
6. Click on $x$ in the Function Calculator (or type $x$ from your keyboard). Click OK.
7. Select the plot you just created and the independent point (whose coordinates you measured). Then choose **Merge Point To Function Plot** from the Edit menu. Drag the point along the plot and observe its coordinates.
7. Drag the unit point—the point at (1, 0)—to the right and left and observe how the scale of the coordinate system changes.

8. Release the unit point when the x-axis goes from about –10 to 10.

**Plotting a Family of Curves with Parameters**

Plotting one particular equation, \( y = x \), is all well and good. But the real power of Sketchpad comes when you plot families of equations, such as the family of lines of the form \( y = mx + b \). You’ll start by defining parameters \( m \) and \( b \) and editing the existing function equation to include the new parameters. Then you’ll animate the parameters to see a dynamic representation of this family of lines.

9. Choose **New Parameter** from the Graph menu. Enter \( m \) for Name and 2 for Value and click OK.

10. Use the same technique to create a parameter \( b \) with the value –1.

11. Select the function equation \( f(x) = x \) (select the equation itself, not its plot) and choose **Edit Function** from the Edit menu.

12. Edit the function to be \( f(x) = m \cdot x + b \). (Click on the parameters \( m \) and \( b \) in the sketch to enter them. Use * from the Function Calculator or keyboard [Shift+8] for multiplication.) Click OK.

13. Change \( m \) and \( b \) (by double-clicking them) to explore several different graphs in the form \( y = mx + b \), such as \( y = 5x + 2 \), \( y = -1x - 7 \), and \( y = 0.5x \). You can learn a lot by changing the parameters manually, as you did in the previous step. But it can be especially revealing to watch the plot as its parameters change smoothly or in steps.

14. Deselect all objects. Then select the parameter equation for \( m \) and choose **Animate Parameter** from the Display menu. What happens?

15. Press the Stop button to stop the animation.

16. Select \( m \) and choose **Properties** from the Edit menu. Go to the Parameter panel and change the settings so they resemble those at right. Click OK.

17. Again choose **Animate Parameter**.

18. Continue experimenting with parameter animation. You might try, among other things, animating both parameters simultaneously or tracing the line as it moves in the plane.

**Presenter:** Stop here and answer questions. You may wish to demonstrate modeling \( y = mx + b \) using sliders instead of parameters (perhaps using sliders from the sample sketch **Sliders.gsp**). Allow about 15 minutes for the remainder of the activity.
Functions in a Circle

How does the radius of a circle relate to its circumference? To its area? These are examples of geometric relationships that can also be thought of as functions and studied algebraically.

You’ll start by constructing a circle whose radius adjusts continuously along a straight path.

19. In a new sketch, use the Ray tool to construct a horizontal ray.

20. With the ray still selected, choose Point On Ray from the Construct menu.

21. With the Arrow tool, click in blank space to deselect all objects. Then select, in order, the ray’s endpoint and the point constructed in the previous step. Choose Circle By Center+Point from the Construct menu.

22. Measure the circle’s radius, circumference, and area.

23. Drag the circle’s radius point and watch the measurements change.

Next we’ll explore these measurements by plotting.

24. Select, in order, the radius measurement and the circumference measurement. Choose Plot As (x, y) from the Graph menu. Can you see your point? If not, it might be off the screen.

25. Choose Rectangular Grid from the Graph | Grid Form submenu. Drag the new unit point at (0, 1) down until you can see the plotted point from the previous step.

26. Select the plotted point and choose Trace Plotted Point from the Display menu. Drag the radius point and observe the trace.

In situations such as this (where you trace something as a point moves along a path) you can often get a smoother picture by creating a locus.

27. Select the plotted point and the radius point; then choose Locus from the Construct menu. What is the slope of the line containing this locus?

28. Repeat steps 24 and 27, except this time explore the relationship between the radius and area measurements. How do the two curves compare? For what radius do the numerical values of a circle’s circumference and area equal each other (ignoring units)?

Further Challenges

- Graph pairs of parallel lines and show that their slopes are the same.
- Graph pairs of perpendicular lines and show that their slopes are negative reciprocals.
- Graph a parabola of the form $y = a(x - h)^2 + k$, using parameters for $a$, $h$, and $k$. 