The Undergraduate Mathematics Teaching Project (UMTP) is a one-year study aiming to characterize, and identify issues related to, mathematics teaching in undergraduate tutorials. It builds on earlier research into mathematics learning in undergraduate tutorials and involves a research collaboration between mathematics educators and mathematicians. Research is ongoing, but issues and indications from analysis of pilot data are included.

Rationale

The broad aim of the Undergraduate Mathematics Teaching Project (UMTP) is to explore, in a collaboration between mathematics educators and university mathematics teachers, current thinking and practices in mathematics teaching at first year undergraduate level. It seeks to elicit relationships between the enacted teaching, the mathematics being taught, the aims and objectives for students’ learning, and the perceptions of those teaching (the tutors) and those observing (the researchers). It will begin to provide a knowledge base on which to make decisions affecting practice in university teaching and illuminate an under-explored area of influence on mathematics teaching more widely.

The research develops from two previous studies. The first explored undergraduates’ mathematical learning difficulties in first year tutorials (Nardi, 1996) – in particular students’ appreciation of abstraction and formalism. The second study followed from the first: data, in the form of transcripts of tutor-student dialogues, and analyses were presented to the tutors (whose tutorials were observed in the first study) to explore their related thinking and reactions (Nardi, 1998). Both studies provided strong evidence of the potential of tutorials as a source of rich data, allowing insights into learning and teaching.

Theoretical perspectives

The research is embedded in a growing theoretical area which focuses on the development of knowledge in advanced mathematics, and the difficulties students face in dealing with mathematical abstraction. The work in the area of Advanced Mathematical Thinking (e.g. Tall, 1991) is highly relevant and the work of Nardi, quoted above, fits into this tradition. The current study seeks to relate these theoretical perspectives to issues in teaching.

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There is an extensive, curriculum-based, literature in this area, mainly in North America, which seeks to relate undergraduate learning to methods of teaching. For example, alternative approaches to calculus (e.g. Ferrini-Mundy & Graham, 1991) and linear algebra (e.g. Leron & Dubinsky 1995) reflect, in part, attempts to make these subjects more engaging and meaningful for the majority of students. However a general perception remains that the teaching of mathematics at the undergraduate level has not to date made sufficient effort to deal with the backgrounds and needs of present day students. The research described here aims to go beyond particular practices, to seek more general awarenesses and understandings of the relationships between teaching and learning at undergraduate level. It draws also on research at other levels of mathematics teaching, seeking commonalities and differences. For example, it explores the use of the Teaching Triad, used to characterize and analyze mathematics teaching at secondary level (Jaworski, 1994).

Project Initiation and Methodology

The UMTP methodology might be described as critically qualitative with a strong emphasis on participation (Jaworski, 1994; 1998). Participants are university researchers (3) and mathematics tutors (6). Qualitative data is sought to allow access to the complexities of tutors' epistemology and its relation to pedagogy. The critical nature of the research is in its questioning of processes and practices at all levels. Questions from researchers to tutors seek access to tutors' thinking underpinning observed actions in tutorials. The earlier research suggested that such questions will lead to questioning by the tutors of their own practices and associated theories and beliefs. Periodic meetings between all the research participants are designed to encourage an airing of tensions and issues. As part of the analysis, feeding back into data collection, researchers will reflect critically on questions asked, to examine and expose their relation to underlying theoretical perspectives, and influences on tutors' responses.

Analysis during data-collection involves scrutiny and annotation of observation notes to produce observation-protocols and suggest questions for interviews. Analysis after data-collection involves scrutiny of recorded interviews and production of interview-protocols for use in extracting characteristics and issues. Protocols act as second-order data to inform subsequent analysis which involves a critical scrutiny of observations, interviews and discussions to elicit relationships. Discussions in research group meetings are recorded and support the analysis of the interviews. Characterization involves seeking processes, practices and issues which might be seen to be germane to a larger number and wider variety of settings. Rigour is to be ensured through triangulation between alternative data sources, and a transparency of contextualization and critique (Delamont and Hamilton, 1984; Ball, 1990).
Indications and issues arising from pilot-study analysis

The pilot study focused on 7 tutors, which would reduce to 6 for the main study. The purpose of the pilot was to try out observational and interview approaches, data gathering techniques and analytical procedures; to induct tutors into research practices; and to enable the researchers to adjust, critically, the mutuality of their perceptions of the research methodology. We offer some insights into the emergent data and our preliminary analysis of it.

Analysis involves critical scrutiny of the data, seeking links to other data items, and recognizing patterns which might be judged characteristic. In the pilot study, only two tutorials were observed from each tutor. Thus, pattern seeking is at a very early and elementary stage. Nevertheless there are analytical remarks to make about these data.

Tutor-1: Data and Preliminary Analysis

We provide, as exemplars here, three points, from analyses of data from one tutor, which may be indicative of forthcoming characterization.

1. Teaching strategies:

The tutor refers to various strategies he uses in his teaching. For example, he used a (white)board to facilitate communication between tutor and students. He emphasized the importance of “explanation skills”. He values “teaching by example”, offering reasons for providing his own examples:

T1. I like teaching by example, especially when you've got something abstract to say and certainly I think ... I consider it quite important. The reason I like to give my own examples sometimes is you quite often get the student who's done the ones on the sheets and a student who hasn't done the ones on the sheet, so you don't want particularly to set the examples on the sheet.

‘Use of strategies’ might become a characteristic. It will be important in other data to look out for tutors' references to strategies they use. In this case, one example of a strategy is the tutor's use of 'his own examples' to provide alternative experience from that offered by the lecturer, and for discussion purposes in the tutorial. With this tutor it will be important to explore further his construction and use of such examples.

2. Developing students’ confidence:

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2 He refers to the lectures which are open to all students, and the problem sheets, set by the lecturers, for students to tackle questions related to the lecture material.

3 Quotations are edited slightly to take out hesitations and repetitions, unless these seem crucial to the sense with respect to the analytical point being made. Analysis, however, looks carefully at the whole data and considers the relevance and import of pausing, hesitation etc.
In parts, Tutor 1 was critical of his own teaching: for example, suggesting that he was “going too fast when student K was struggling. He felt it very important to try to develop students’ confidence:

T1. I think the biggest thing to developing confidence is to make them not feel at all uneasy about being wrong. I mean …you don't have to have a wonderful rapport but just enough of a …never be patronizing and you know, just, er, OK, OK, tell them the question's hard, whether it is or not because that's sticking with them, tell them it's hard and [as] such it's, it's just a matter of er, getting your, your confidence to, to, to be wrong, you know is is a very good thing.

Some of the hesitation is left in here to indicate the tutor's struggle to articulate his perception of dealing with students' confidence. This statement points towards a 'sensitivity to students' on the part of the tutor – a teaching concern directed at the students' needs, in this case in the affective domain. It suggests that doing what he can to encourage confidence is going to help the student to tackle concepts more effectively.

He also suggested that he needed to be more sure of students’ basic understanding, On one occasion he felt he had produced a “poor” diagram [It represented three vectors in the plane, but might have been seen as a set of three orthogonal axes in $\mathbb{R}^3$.] He said, “I should have been more careful …”. This criticism might be seen as sensitivity which is more cognitively focused: he suggested the diagram might have been construed in an alternative way to that intended, potentially leading to misunderstanding by the student.

Sensitivity to Students (SS) is one of the elements of the Teaching Triad (Jaworski, 1994, Jaworski & Potari, 1998), the others being Management of Learning (ML) and Mathematical Challenge (MC). The teaching triad research suggests that SS is closely allied to MC which characterizes a teacher’s interaction with the student in the domain of mathematics, attempting to encourage the student's cognitive development. The tutor's use of the diagram could be seen as a means of challenging students, which was unsuccessful because of the inadequacy of the diagram. Thus there are indications in this very early data of the inter-relationship of SS and MC leading to issues about teaching for students’ conceptual development.

2. Ensuring Rigour:

Tutor 1 recognized his emphasis on the importance of “the formalizing process”, or “proof”:

T1 Yes. Well, I mean (pause). My role is to have them… I mean my role is an important one is that the, er, I am their only source of rigour that they have in the, really in the, in the, the first year. They'll, it, it's me explaining things and introducing style and er proof. But at the same time ideas I'd like to get er, it's it's a balance between getting ideas right, er, and giving them some feel for them but then
also being able, being somehow being able to (pause) convince them that they feel they have to do er... concept is the same as the rigorous definition that might be on the board. …

I think, I think the important role of the tutor is to make sure the, try and give them handle on both things but that's often difficult. I mean one way is usally geometric and one way is analytic and a lot of people only think one way and not the other but er, at the same time I usually tell them as long as they can do it one way it's a good thing but it's always nice to have two ways of thinking about it. ...

Scrutiny of the transcript shows that this statement is one of Tutor 1's longest statements in the interview. Although very hesitant, perhaps unused to articulating his personal principles or theories, he presents a clear rationale for his emphasis on, and approaches to, rigour. This length and clarity are interpreted as indicating issues of significance to the tutor – in particular, perhaps, the issue of "getting ideas right" versus "giving [students] some feel for them" (our emphasis).

An emerging issue, the nature and importance of interaction

Interaction in tutorials, unsurprisingly, is perceived and enacted in different ways by different tutors. Many students are typically silent unless asked to speak, and so, to some tutors, their uttering a word indicates interaction. In some cases the interaction is little more than a question from a student, followed by a monologue by the tutor. In others it involves a student in presenting ideas to the tutorial group, with comments and questions from other student(s) and tutor. Some tutors seem to see interaction as little more than an unfortunate necessity for discovering what is problematic, especially with weaker students. Others see it as a tool for empowering students' mathematical communication.

From the current data, we have identified elements of tutor ‘inciting’ interaction and ‘empowering’ interaction. The following quotation (from Tutor 34) emphasizes the belief that interaction is a necessity for helping weaker students – "I very often ask them to explain it to me" – but unnecessary for "rather better" students where a monologue suffices.

T3 Um, I think it's actually more to do with the students ... because they are very weak. In trying to find out whether they understand something I very often ask them to explain it to me or say something about it. Because, er if I just say OK this is a Cauchy sequence and they nod I don't actually know whether they're just nodding to just to be polite ... Er, my Monday afternoon students are, are rather better and tutorials tend to be much more of a monologue. Um, I don't, I don't need to ask them what a Cauchy sequence is because they've demonstrated in their work that they know.

4 There are no quotations from tutors T2, or T5-7
Using tutorials to empower students has emerged from some of the current data. For example, Tutor 1, above, spoke of the importance of building students' confidence, for example, of acknowledging that certain concepts are "hard", and of seeing "being wrong" in positive terms. Although these ideas were not expressed explicitly in terms of interaction, it might be believed that this tutor would engage his students in dialogue to expose "wrong" perceptions, to cut through hard mathematical concepts.

Another tutor (Tutor 4) talks explicitly of using dialogue for empowerment in students' learning "how to do mathematics" – "I want them to actually be doing it rather than me telling them how to do it”.

T4 I think I view it that the point of a tutorial is for them to learn how to do mathematics so I want them to actually be doing it rather than me telling them how to do it because I think they learn less by me telling them the answer to a dozen questions instead of, OK, how do I think of these solutions, how do you think of these solutions? How do you think of these solutions now and when you're back in your room without me prompting you how to do it?

In Conclusion

It is important to emphasize that we have as yet only a small set of data to analyze, so that the issues indicated above only just start to suggest characteristics which might be seen as more widely germane. However, the pilot has fulfilled its purpose in clarifying methodology and establishing initial relationships, quite apart from its promise regarding issues and characteristics. We look forward to presenting results from our main study at the conference.

References


